DOI: 10.15587/978-617-8360-09-2.CH3

Bogdana Gayvas, Yevhen Chaplya, Veronika Dmytruk,
Bogdan Markovych, Anatolii Dmytruk

CHAPTER 3

MATHEMATICAL MODELING OF DRYING PROCESSES IN POROUS MATERIALS CONSIDERING CAPILLARY PROPERTIES

ABSTRACT

The Chapter aims to provide a detailed understanding of the drying kinetics and to identify conditions under which drying is most efficient, considering external factors such as airflow and electric fields. The model aims to predict the distribution of liquid and gas phases within the porous structure and the resulting mechanical stresses, contributing to optimizing drying processes in industrial applications. A particular focus is on the capillary properties of the porous medium being dried. Moreover, a sustainable mathematical model is proposed for analyzing the moisture and temperature distribution, radial displacements, and stresses within a multicomponent dispersed material of the capillary-porous structure. By solving the key system of differential equations for mass and heat transfer, and incorporating the mechanical properties of the material, the model predicts the changes in moisture concentration, temperature, and mechanical stresses in a material at any point in time and its placement within the layer. The work results provide insights into the drying kinetics and mechanical behavior of the grain under different drying conditions.

KEYWORDS

Drying, porous materials, capillary properties, phase transition, mass transfer, diffusion, drying kinetics, mathematical modeling, structural model.

In modern conditions, sustainability is the primary requirement for developing all technologies used in the national economy. The field of agricultural product processing, specifically drying, is no exception. Since drying is an extremely energy-intensive process, one of the main requirements for its implementation is saving energy resources. Therefore, engineering tasks aimed at reducing energy consumption in the drying of agricultural products are urgent and relevant.

The application of mathematical modeling is one of the effective ways to solve engineering problems due to the accessibility of its tools and their powerful capabilities. Specifically, mathematical modeling is widely used to develop new drying technologies.

The problem under consideration is the convective-heat drying of a layer of wet dispersed material with a capillary-porous structure. It is possible to consider the stage of falling drying, when the grain is already heated to the desired temperature and is dried for a certain period of time. The heating of the drying agent stops, i.e., the grain is blown by the drying agent of an ambient temperature. This will provide an opportunity for crucial savings on energy resources.

Models of drying are typically based on the thermodynamics of irreversible processes. The hypothesis of local thermodynamic equilibrium is accepted, which, for small deviations from the equilibrium position in continuous media, is justified in the works of S. de Groot, I. D'yarmati, A. Kovalenko, and others.

Along with internal energy, the existence of a state function, entropy, is postulated. The concept of entropy is introduced to distinguish between reversible and irreversible processes. The acceptance of the hypothesis of local equilibrium, considering the equations of state, entropy balance, energy, mass, and momentum in many cases allows for the determination of all parameters characterizing the irreversible process.

Drying models must consider the actual mechanisms of heat and moisture transfer depending on the drying method, the impact of kinetic and geometric characteristics of the body, the controlling parameters of the drying agent, and their impact on the stress-strain state and stability of body shapes, maintaining their quality during the drying process. They are based on specific macroscopic physico-mathematical models of heat and mass transfer and require effective analytical-numerical methods for solving the corresponding boundary value problems of mathematical physics.

For the physico-mathematical modeling of processes in solid bodies, thermodynamic methods have been developed in the works of Y. Burak, E. Chaplya, O. Chernukha, and others [1]. Special attention is required for the mathematical description of the material structure. From the perspective of significance for the agro-industrial and food sectors of Ukraine's economy, special attention is deserved by the modeling of drying processes of dispersed bodies with capillary-porous structures, which are multiphase and heterogeneous.

The mathematical model of drying capillary-porous bodies, as a special case of the heat and mass transfer model in an n-component three-phase deformable porous moist medium with phase transitions and chemical reactions, studies the evolution of temperature, moisture content, pressure, kinematic characteristics of the process, sensitivity to parameter influences and boundary conditions. A moist porous body is generally considered as a three-phase medium containing a skeleton (porous or granular structure), liquid, and gas (homogeneous steam-air mixture), filling the pores or gaps between the grains, although the material is generally treated with chemically active mixtures before drying, interacting both among themselves and with the material, forming n-phase structures.

For reflecting multiphase nature, approaches of mixture theory and methods of multi-velocity system mechanics, capillary models of the porous body, and combined methods are used. When modeling heat and mass transfer processes, capillary models are used, and when solving mechanics problems, methods of homogenizing the heterogeneous structure are employed, obtaining physical relationships for the body as a whole based on certain assumptions about the nature of temporal and spatial changes in the studied fields.

The continuum-thermodynamic approach to building physico-mathematical models of solid solution mechanics, describing interconnected mechanical, thermal, and diffusion processes, considering local state changes of the components, was developed in the works of Y. Burak, E. Chaplya, O. Chernukha, B. Gayvas, B. Gera, and others. This approach is based on the following propositions: for spatially inhomogeneous and non-equilibrium systems, the hypothesis of local thermodynamic equilibrium is accepted, implying that the state of physically small subregions of the system is determined by conjugated physical parameters describing the equilibrium state. For describing mechanical, physical, and chemical processes, the conjugated parameters are pressure and volume; stress tensor and strain tensor; thermal conductivity – absolute temperature and entropy; diffusion – chemical potential and particle concentration. To determine the change in mass, energy, and momentum, balance and kinetic relationships and equations of state are used.

The description of interconnected thermal, mechanical, and diffusion processes in porous multi-continuum media was considered by B. Gayvas, O. Hachkevych, L. Khoroshun, Y. Kubik, R. Kushnir, R. Terlets'kyi, V. Chekurin and other scientists. If the characteristic distances over which the system parameters change are greater than the characteristic dimensions of the inhomogeneities, the principle of homogenizing the heterogeneous structure is used. Then the macroscopic parameters are taken as the average effective coefficients over elementary volumes, satisfying the classical equations of thermoelasticity. The effective coefficients for granular, fibrous, and layered media were determined based on stochastic equations for micro-inhomogeneous bodies.

For building models of finely dispersed media, the approach of multi-phase media mechanics is also used, developed in the works of A. Neimark, L. Heifets, R. Nigmatulin, V. Nikolaevsky, and other scientists.

Mathematical models of drying porous bodies are based on the laws of conservation of mass, momentum, and energy, as well as known experimental dependencies, on the basis of which equations of heat and mass transfer of moisture in the body are constructed. The possibility of quantitative estimation of heat passing inside the body due to thermal conductivity is based on Fourier's hypothesis. At the same time, diffusion flows are considered based on Fick's laws, and filtration flows on Darcy's law. The similarity of heat and mass transfer processes is considered in the works of A. Kutz, V. Mustyats, M. Razin, and others. When building models of convective mass transfer, both conditions of convective mass exchange and conjugated problems using boundary layer theory are considered. In addition, the drying model includes the basics of the theory of thermoelasticity of viscoelastic bodies, considering the anisotropy of the structure and the basics of the theory of shape stability [2].

The formulation of drying problems is based on models describing interconnected phenomena and processes of heat and mass transfer and deformation occurring during drying. The drying process is not only a thermophysical one but also a technological one, where the forms of moisture bonding with the material, the regularities of heat and moisture transfer in wet materials during the interaction of porous bodies with hot surfaces, heated gases, and electric fields play a significant role. During drying, the technological, biological, and structural-mechanical properties of the material change, due to the change in the forms of moisture bonding with the material and its partial removal by evaporation. The science of the forms of moisture bonding with the material was developed by P. Rebinder and further developed in the works of S. Lipatov, G. Maksimov.

P. Kostrobij's research in the field of thermodynamic potentials, diffusion processes, and simulation of interactions in a semi-infinite medium is related to the mechanisms of heat and mass transfer and reaction-diffusion processes in porous materials during drying and can be successfully applied to such problems. B. Markovych, M. Tokarchuk, I. Ryzha and others suggeste using non-integer integral-differentiation to model systems, which are characterized by "memory" effects, structural heterogeneity, spatial non-locality, deterministic chaos, and self-organization.

A large number of works are devoted to the mathematical description and quantitative study of drying porous materials in stationary and non-stationary modes. Among them, the works of A. Bomba, N. Grinchik, O. Lykov, P. Lutsyk, Y. Sokolovsky, describe the processes in porous media based on the principle of homogenizing the heterogeneous structure of the body. The basics of this theory for describing moisture transfer within the material are formulated based on the classical theory of diffusion by P. Kosovich, A. Lebedev, Y. Miniovich, U. Lewis, and T. Sherwood.

In his research, O. Lykov introduced the concept of mass transfer potential, which serves as the driving force for the fluid flow under isothermal conditions and is based on experimental laws of heat and moisture conductivity [3]. The quasi-homogeneous approximation was used to analyze transfer processes, assuming the replacement of the real dispersed medium with a continuous medium with effective coefficients. The heat and moisture transfer equations included empirical coefficients dependent on temperature and pressure. Drying processes of capillary-porous bodies were studied within the framework of the single-continuum approach and based on the thermodynamics of irreversible processes. For modeling the vapor source, the phase transition criterion was introduced. This parameter is generally considered a function of moisture content but depends on all process parameters and can be arbitrarily assigned by different authors.

Also, O. Lykov obtained a system of differential equations of heat and mass transfer in capillary-porous materials in the region of hygroscopic moisture content. In particular, it is assumed that the non-equilibrium state of the system at any moment of the drying process is caused by the inhomogeneous distribution of temperature, moisture content, excess pressure, and deformations. The increase in entropy in systems is associated with irreversible processes of thermal conductivity, molecular and convective diffusion, phase transition, moisture-thermal deformation, and irreversible cross-processes (Dufour and Soret effects).

T. Sherwood proposed the hypothesis of deepening the surface of moisture evaporation during drying. The walls of the porous body skeleton absorb water, causing the body to swell when moistened and shrink when dried. To simplify the analysis of transfer processes in finely dispersed bodies, it is advisable to use averaged characteristics, whose use in models, as experiments show, gives results close to measured ones. The optimal control for the element functional properties under heating and the methodology for predicting its behavior is realized in [4].

In the views on the mathematical modeling of heat and mass transfer in the non-stationary case, significant differences arose among scientists. For example, L. Heifets and A. Neimark believed that the formal introduction of the mass transfer potential obscures the real mechanisms of mass transfer [5]. Since the relationship between the evaporated liquid flow and the specific saturation is nonlinear, determined by geometric characteristics, the use of thermodynamics of irreversible processes and Onsager's relationships, which assume a linear dependence of flows on thermodynamic forces, is unfounded. Their mathematical modeling methods for the drying process are based on the description of multiphase processes in porous media. The mutual distribution of phases in the pore space is one of the main characteristics of multiphase heterogeneous processes in porous media. In the case of processes with phase transitions, the mutual distribution of phases is established as a result of one phase transitioning into another and the redistribution of phases within the porous body due to the difference in capillary properties.

Recently, for modeling filtration processes in porous media, it has been proposed to use differential equations with fractional-order derivatives both in time and coordinates (O. Lopatiev, Y. Pyanylo, A. Torskyy and others) [6].

A large number of works are devoted to the interaction of the body with the drying agent. Heat and mass exchange between the material being dried and the environment (drying agent) is one of the determining factors of the process. Heat exchange coefficients can be obtained directly from experiments, analytically using boundary layer theory and statistical methods, as well as from criterion equations.

The heat exchange coefficient during forced movement of the environment is obtained from the Nusselt criterion based on the criterion equation $Nu = f(Re, Pr)$, constructed empirically from experimental data.

Experimental studies by I. Fedorov, F. Polonskaya, and others have shown that the heat exchange coefficient in the drying process is greater than the pure heat exchange coefficient under the same conditions. In works of O. Lykov, P. Lebedev, V. Frolov and G. Shubin, it is shown that the heat exchange coefficient is constant only during the period of constant drying rate. In the period of falling drying rate, it continuously decreases, approaching the value of dry body heat exchange.

An important component of optimization the modeling drying processes is determining the effect of porosity change on the stress state of the body [7]. The stress-strain state of the body can change, and the body can lose its stability under the action of various physico-chemical processes (mechanical action, drying, phase transitions, etc.). Due to the need for a more detailed study of the mutual influences of stresses on mass transfer and mass transfer on stresses in a porous layer, many studies are conducted. It is considered appropriate when describing mass transfer processes to proceed from capillary models of the porous body, and when describing strength characteristics from homogenized models considering distortion caused by heat and mass transfer processes. Inhomogeneous deformations lead to the appearance of residual stresses, which are superimposed on the stresses caused by mechanical actions and can cause instability of the equilibrium shape.

3.1 MATHEMATICAL MODELING OF THE DRYING PROCESS

Mathematical modeling of the drying process is based on the selection of a model for the structure of the porous material; taking into account the specifics of multiphase mass transfer (interaction of phases with each other; considering phase and capillary forces that determine the mutual distribution of phases in the elementary physical volume of the porous body on one hand, and on the other are the driving forces of the process); formulation of boundary value problems that reflect the interaction of the external environment with the porous material under characteristic methods of drying intensification.

The movement of moisture under the influence of temperature (thermo-moisture conductivity) includes phenomena [3]:

– molecular diffusion of moisture, in the form of molecular leakage of steam, which occurs due to different velocities of molecules in the heated and cold layers of the material;

– capillary conductivity, due to the change in capillary potential, which depends on surface tension, which decreases with increasing temperature, and since the capillary pressure over the concave meniscus is negative, the decrease in pressure increases the suction force, resulting in moisture in the form of liquid moving from the heated layers of the body to the colder ones;

– movement of liquid in the porous body in the direction of the heat flow caused by the presence of trapped air. When the material is heated, the pressure of the trapped air increases and air bubbles expand. As a result, the liquid in the capillary pore is pushed in the direction of the heat flow (trapped air pushes the liquid to layers with a lower temperature).

The influence of trapped air, porosity, temperature, and material saturation with moisture on the process of conductive drying of a capillary-porous body is considered in [8, 9].

Since real porous materials have an irregular and random structure, stochastic structural models can claim adequacy [10, 11]. Statistical analysis of the distribution of liquid and gas in hydrophilic porous media is based on representing the pore space as a system of channels with variable cross-sections, which create a stochastic spatial lattice with a certain coordination number [5]. This representation allows the problem of liquid and gas distribution to be reduced to the problem of the mutual distribution of liquid and gas pores in a random lattice with certain statistical properties. Using the apparatus of penetration theory (percolation), it is shown that in the pore space during drying, moisture forms three characteristic configurations: a connected system of gas-filled pores that exits to the outer surface; a two-phase system (a set of unconnected liquid inclusions blocked by gas); and a liquid pore system. For each specific porous structure, there is a critical moisture content, at which the connectivity of the liquid phase is completely disrupted, and all moisture is localized in isolated inclusions. The value of the critical moisture content is a structural characteristic of the porous medium and can be used for a comparative analysis of different porous structures [12].

The mutual distribution of phases in the pore space has a significant impact on the processes occurring in the body. It determines the effective transfer coefficients across phases, the magnitudes of interphase surfaces [13]. Capillary forces are responsible for the phase distribution.

Changes in the liquid content in the porous body occur through liquid evaporation and capillary absorption [5]. The heat and mass transfer processes occurring in the pores, the properties of liquids and gases in the pores, and the methods of their modeling are covered in [13]. Experimental studies of diffusion, capillary, and film transfer mechanisms [5] indicate the decisive influence of capillary and surface forces on the mass transfer process and drying intensity. An overview of research results on heat and mass transfer in porous media is provided in [13, 14].

To account for the influence of geometric factors, it is possible to use a statistical consideration of the process in an elementary physical volume based on a certain structural-metric model. One type of capillary model is systems of parallel capillaries of different radii. The statistical average can be represented using the distribution function. The capillary radius distribution function correlates with the functions of pore size distribution measured by various methods. This correlates with the method of studying the filtration of cement stone, wood, and other porous materials based on their averaged characteristics, which allows not to account for fluid movement along winding paths and interflows and to consider the porous body as a material with homogeneous properties for the chosen filtration direction. The reproduction of the mass transfer pattern using capillary models based on the description of the sizes and shapes of pores in porous bodies is proposed in work [5].

3.1.1 HEAT AND MASS TRANSFER WITH THE EXTERNAL ENVIRONMENT

A significant number of works are devoted to the interaction of the body with the drying agent. Heat and mass transfer between the dried material and the medium (drying agent) is one of the determining factors of the process. The heat transfer coefficients can be obtained directly from experiments, analytically using the boundary layer theory [15], and statistical methods, as well as from criterial equations [3]. The boundary layer theory [15] provides solutions to various problems of the hydrodynamic boundary layer. Based on experimental studies, N. Mikheev and B. Smolsky indicated the influence of internal mass transfer on the field of partial pressures at the beginning of the drying process and concluded that the heat transfer coefficient must be calculated based on the solution of the conjugation problem.

To generalize experimental data when describing the drying process, the theory of similarity is also used, which allows combining physical quantities that characterize the process into dimensionless complexes. The obtained criteria are considered as new variables that reflect the influence of both individual factors and their combination [3].

The heat transfer coefficient during forced movement of the surrounding medium is obtained from the Nusselt criterion based on the empirical criterial equation constructed from experimental data.

Experimental studies by I. Fedorov, F. Polonskaya, and others showed that the heat transfer coefficient during drying is higher than the pure heat transfer coefficient under the same conditions. In the works of O. Lykov, P. Lebedev, and G. Shubin, it is shown that the heat transfer coefficient is constant only during the period of constant drying rate. During the period of falling drying rate, it continuously decreases, approaching the heat transfer value of the dry body. The driving force of moisture exchange during moisture evaporation is considered to be the difference in partial pressures between the surface layer and the surrounding medium, although in the environment of superheated steam, the difference in partial pressures is absent. O. Lykov believed that molecular-molecular mass exchange on the surface is determined by the gradient of the chemical potential and the total pressure. Calculations in this case showed that the necessary mass transfer can be ensured only under the condition of a slight overheating of the surface of the wet sample relative to the saturation temperature at a given pressure.

There is also an opinion that mass exchange during drying in a stationary environment is determined by the chemical potential difference between the surface of the body and the heat carrier. This explains the significance of the medium temperature, higher than 1400 °C, if drying of thin material in the environment of superheated steam exceeds the drying rate in the air environment. Experiments on geometrically identical samples with different initial moisture contents at a constant Reynolds number showed a decrease in the heat transfer coefficient with decreasing initial moisture content. Reducing the hydraulic radius increases the heat transfer coefficient. To determine the mass transfer coefficient, it is considered advisable to use the analogy between heat transfer and mass transfer.

Within the framework of the capillary approach, for porous bodies with small dispersion of the transverse dimensions of pores in the quasi-stationary approximation, complete systems of relations for describing the processes of natural or stimulated by blowing or external constant electric field drying are formulated.

In the works [16, 17], the problems of isothermal two-sided drying of an initially moisture-saturated porous layer under the action of an external constant electric field are solved by adopting the model of cylindrical capillaries of a porous material. The law of phase boundary movement in time is determined for problems of enhancing natural two-sided drying of a porous layer, and the influence of electroosmosis on the drying of a porous layer under both natural and convective drying of one of the surfaces is investigated. Quantitative studies of the change in relative humidity over time due to the magnitude of the electric field intensity are conducted.

It is shown that with the increase in the intensity of the electric field, the drying process intensifies. The dependence of the critical time and the corresponding moisture content on the characteristics of the material and the intensity of the electric field is established. The curves of the relative moisture content over time during two-sided drying have a characteristic kink. Quantitative studies of the dynamics of the relative humidity of the layer indicate a reduction in drying time compared to the case when electroosmosis is absent.

In the work [19], a system of equations for the electrodynamics of a hereditary electroconductive nonferromagnetic porous body, taking into account the dependence of its characteristics on moisture and temperature, is formulated. The problem of high-frequency heating for such bodies is considered. The dependence of the heating intensity on the distance from the electrodes to the surface of the body being heated is studied. The absorption of heat and moisture by the solid body leads to an increase in its volume, which, being non-uniform, creates stress. In particular, cyclic changes in the external conditions in which the material is located create variable stresses that can lead to the growth of defects of dangerous sizes.

Advancements in science and engineering have been significantly driven by the observed similarities between heat and mass transfer processes and the universality of diffusion equations. By leveraging common principles and mathematical descriptions, scientists and engineers can develop more efficient and effective solutions across a wide range of applications.

Recent investigations into complex systems have utilized heat and mass transfer models to achieve various outcomes. For instance, in [21] there is proposed an innovative approach for image restoration and contrast enhancement using a nonlinear reaction-diffusion model. This model simulates the behavior of a heat equation in low-gradient areas while halting diffusion in high-gradient regions to preserve edges. Their algorithm employs a divide-and-conquer technique coupled with a reaction-diffusion system. Similarly, a novel numerical approach using the Lattice Boltzmann method for a Gray-Scott-based reaction-diffusion model was introduced in [22], effectively handling noisy images by comparing pixel motion to fluid motion.

Further contributions include [23], which proposed a spatio-temporal dynamics model for a tritrophic reaction-diffusion system, offering methodologies for optimal system control. In [24], the spatio-temporal dynamics using a fractional order SEIR model, relevant for understanding drying processes, are explored. The work [25] developed a nonlinear fractional partial differential equation for image inpainting, applying nonlinear diffusive filters. Stability analysis techniques for reaction-diffusion systems with delay, essential for drying process models, are provided in [26]. In [27], a diffusive phytoplankton–zooplankton–nanoparticle model with a density-dependent predator death rate is formulated, as well as the analysis of its local stability. A time-fractional diffusion equation for image and signal smoothing is used in [28], demonstrating the use of anomalous diffusion behaviors in image processing.

In the context of catalytic reactions, in [29] it is provided a statistical description of catalytic hydrogen oxidation, incorporating both the diffusion of reactants and the magnetic properties of ions and atoms involved. A generalized Cattaneo-type diffusion equation in time-fractional derivatives was developed for electrons, accounting for the complexity of relaxation electromagnetic diffusion processes in layered nanostructures in [30]. In [31], it is suggested a discrete solution for nonlinear parabolic equations with diffusion terms, proving the existence and uniqueness of weak solutions using an internal approximation combined with the backward Euler scheme. A DDFV scheme for nonlinear parabolic reaction-diffusion problems on general meshes, applicable to complex geometries in capillary-porous structures is developed in [32]. In [16], solving Stefan's linear problem for drying cylindrical timber is addressed, proposing solutions for phase change problems in drying.

In [33], identification of diffusion scattering parameters for a modified viral infection model is proposed, while in [34] the examination of the dynamics of a diffusive SARS-CoV-2 model using fractional Laplacian operators is carried out. In [35], kinetic and hydrodynamic approaches in the theory of dense gases and liquids far from equilibrium are unified, under arbitrary Knudsen number conditions, with insights into liquid and gas interactions. In [36], it is discussed coupled compressible two-phase flow, providing further insights into these interactions. In [37], focus is made on impurity diffusion processes, essential for accurately describing mass transfer equations.

In finance, it is proposed a penalty approach for pricing the American-style Asian option under the Merton model, capturing the skewness and kurtosis features of return distributions [38]. In [39], the diffusion of money income, providing methodologies for solving financial diffusion problems, is modeled. Calcium profiles in neuronal cells are studied in [40], allowing insights into the dependency of calcium concentration on variable diffusion coefficients.

The problem of optimizing the drying time of porous bodies is still relevant, determining the most efficient operation mode of the drying installation [7]. The optimization problem formulation includes a functional that should be minimized and a set of admissible solutions. As a rule, the objective function is a function of many variables. The objective functional is minimized on a set from the functional space. Given the environmental parameters, based on the solution of the direct drying problem of an initially moisture-saturated porous layer, the problem of minimizing the total drying time by temperature T and pressure P on the layer surface is formulated. The problem is reduced to the problem of minimizing the temperature on the moving phase transition front under certain constraints. The optimal pressure on the layer surface and the minimum temperature on the phase transition boundary that minimize the total drying time are found, given the known parameters of vapor and air densities (or relative humidity) and the temperature of the atmospheric environment, as well as the geometric parameters of the body and the boundary layer. If an analytical solution to the drying problem is found, choosing the necessary optimization criterion, one can optimize the drying regime of the material.

3.1.2 EQUATIONS OF THERMO-HYGRO-MECHANICAL PROCESSES IN A POROUS BODY IN THE HOMOGENEOUS APPROXIMATION (GENERALIZED COORDINATES: TEMPERATURE, DEFORMATION, MOISTURE CONTENT)

Consider a porous body in which the phase transition of liquid to vapor occurs, resulting in a vapor flow. The reduction of moisture in the body and its heating can cause shrinkage and thermal expansion of the material. Additionally, intensive drying generates additional pressure in the vapor-air mixture. Consequently, stresses arise in the body, the magnitudes of which, as studies show, can cause deformation and destruction of the body. In formulating the equations of thermo-mechanical-mass transfer, it is possible to follow the works [3, 11].

Assume that the primary physical processes occurring in the body during drying are mass and heat transfer and deformation. Let's consider these processes in the homogeneous approximation. Under the hypothesis of local equilibrium, the thermomechanical state of a physically small element of the body will be characterized by the parameters: stress tensor $\hat{\sigma}$ – strain tensor $\hat{e}$; temperature T – entropy S; chemical potential μ – moisture content W. The corresponding Gibbs equation for free energy is written as [11] $dF = \hat{\sigma} : d\hat{e} - SdT - \rho_0 \mu dW$, where ρ_0 is the density of the absolutely dry body. The state equations in terms of moisture content are: $\hat{\sigma} = \left.\frac{\partial F}{\partial \hat{e}}\right|_{T,w}$, $S = -\left.\frac{\partial F}{\partial T}\right|_{\hat{e},w}$, $\rho_0 \mu = -\left.\frac{\partial F}{\partial W}\right|_{T,\hat{e}}$.

If the free energy of an elastic isotropic body is represented as a Taylor series expansion in terms of the components of the tensor $\hat{e}$ and retaining terms no higher than second-order smallness [11]:

$$F\left(I_1, I_2, T, W\right) = F_0 + \frac{\partial F}{\partial I_1} I_1 + \frac{\partial F}{\partial I_2} I_2 + \frac{\partial^2 F}{\partial I_1^2} \frac{I_1^2}{2},$$

where $F_0(0,0,T,W)$ is the free energy per unit volume in the absence of deformations; $I_1 = e$ is the first invariant of the strain tensor; $I_2 = e_{ij} e_{ij}$ is the second invariant of the strain tensor, then the components of the stresses can be expressed as:

$$\hat{\sigma} = \frac{\partial F}{\partial I_1} \frac{\partial I_1}{\partial \hat{e}} + \frac{\partial F}{\partial I_2} \frac{\partial I_2}{\partial \hat{e}} + \frac{\partial^2 F}{\partial I_1^2} \frac{\partial I_1}{\partial \hat{e}} I_1,$$

$$S = -\frac{\partial F_0}{\partial T} - \frac{\partial^2 F}{\partial I_1 \partial T} I_1 - \frac{\partial^2 F}{\partial I_2 \partial T} I_2 - \frac{\partial^3 F}{\partial I_1^2 \partial T} \frac{I_1^2}{2},$$

$$\rho_0 \mu = -\frac{\partial F_0}{\partial W} - \frac{\partial^2 F}{\partial W \partial I_1} I_1 - \frac{\partial^2 F}{\partial I_2 \partial W} I_2 - \frac{\partial^3 F}{\partial I_1^2 \partial W} \frac{I_1^2}{2}. \quad (3.1)$$

In the absence of deformations $\hat{e}$ the change in free energy F_0 due to changes in temperature and moisture content and the averaged specific heat capacity in the isochoric process can be represented as follows:

$$F_0 = \int_{T_0}^{T} \left(\frac{\partial F_0}{\partial T} \right)_{W,e=0} dT + \int_{W_0}^{W} \left(\frac{\partial F_0}{\partial W} \right)_{T,e=0} dW,$$

$$\rho_0 C_{W,e} = T \left(\frac{\partial S}{\partial T} \right)_{W,e} = -T \left(\frac{\partial^2 F}{\partial T^2} \right)_{W,e}.$$

Then $\left(\frac{\partial F}{\partial T} \right)_{W,e} = -\int_{T_0}^{T} \frac{\rho_0 C_{W,e}}{T} dT$, where $C_{W,e}$ is the specific heat capacity in the absence of deformation and at constant moisture content.

If U is the internal energy of the body, then in an isothermal-isochoric process ($T = const, \hat{e} = 0$):

$$\left(\frac{\partial F_0}{\partial W}\right)_{T,e=0} = \left(\frac{\partial U}{\partial W}\right)_{T,e} - T\left(\frac{\partial S}{\partial W}\right)_{T,e},$$

where $\left(\frac{\partial U}{\partial W}\right)_{T,e}$ is the heat of the phase transition; $T\left(\frac{\partial S}{\partial W}\right)_{T,e}$ is the heat due to the irreversible transfer of the absorbed substance (Dufo effect).

At low temperatures $T\left(\frac{\partial S}{\partial W}\right)_{T,e} << \left(\frac{\partial U}{\partial W}\right)_{T,e}$, thus $\left(\frac{\partial F_0}{\partial W}\right)_{T,e} = \left(\frac{\partial U}{\partial W}\right)_{T,e} = \rho_0 \varepsilon_\Phi r^*$, where r^* is the specific heat of the phase transition; ε_Φ is the degree of completeness of the phase transition, similar to the phase transformation criterion introduced by O. Lykov [3].

Taking into account that:

$$\frac{\partial F}{\partial I_1} = -K\left[\alpha_{TW}\left(T - T_0\right) + \beta_{TW}\left(W - W_0\right) + \frac{1}{K}\left(P - P_0\right)\right] = -\frac{K}{3}\varepsilon, \quad (3.2)$$

$$\frac{\partial F}{\partial I_2} = G = \frac{E}{2(1+\nu)}, \quad \frac{\partial^2 F}{\partial I_1^2} = \frac{1}{3}(K - 2G) = \lambda = \frac{E\nu}{(1+\nu)(1-2\nu)},$$

where υ is the Poisson's ratio; λ is the Lame coefficient; $E = \frac{(1+\nu)(1-2\nu)}{3\nu}(K - 2G)$ is the Young's modulus; K is the bulk modulus; G is the shear modulus; ε is the resulting change in the unit volume of the body in the absence of stresses; T_0 is the initial temperature; T is the actual temperature; P is the pore pressure; P_0 is the atmospheric pressure; α_{TW} is the coefficient of linear thermal expansion; β_{TW} is the coefficient of linear shrinkage.

The free energy of an elastic isotropic body can now be written as [3]:

$$F\left(\hat{e}, T, w\right) = GI_2 + \frac{1}{6}(K - 2G)e^2 - Ke\left[\alpha_{TW}\left(T - T_0\right) + \beta_{TW}\left(W - W_0\right) + \frac{1}{E}\left(P - P_0\right)\right] -$$
$$-\int_{T_0}^{T} dT \int_{T_0}^{T} \frac{\rho_0 C_{W,e}}{T} dT + \rho_0 r^* \int_{W_0}^{W} \varepsilon_\Phi dW, \quad (3.3)$$

where $e = \vec{\nabla} \cdot \vec{u}$ is the relative volume change of the body; $\vec{u}$ is the displacement vector.

Then the state equations (3.1) take the form [3]:

$$S=-I_2\frac{\partial G}{\partial T}-\frac{e^2}{6}\frac{\partial}{\partial T}\left(K-2G\right)+e\frac{\partial}{\partial T}\left[K\left(\alpha_T\left(T-T_0\right)+\beta_W\left(W-W_0\right)+\frac{1}{K}\left(P-P_0\right)\right)\right]+$$

$$+\int_{T_0}^{T}\frac{\rho_0 C_{W,\varepsilon}}{T}dT+\frac{\partial}{\partial T}\left(\rho_0 r^*\int_{W_0}^{W}\varepsilon_\Phi dw\right); \tag{3.4}$$

$$\rho_0\mu=-I_2\frac{\partial G}{\partial W}-\frac{e^2}{6}\frac{\partial}{\partial W}\left(K-2G\right)+$$

$$+e\frac{\partial}{\partial W}\left[K\left(\alpha_T\left(T-T_0\right)+\beta_w\left(W-W_0\right)+\frac{1}{K}\left(P-P_0\right)\right)\right]-\rho_0\varepsilon_\Phi r^*; \tag{3.5}$$

$$\hat{\sigma}=2G\hat{e}+\frac{1}{3}\left(K-2G\right)e\hat{I}-K\left(\alpha_T\left(T-T_0\right)+\beta_W\left(W-W_0\right)+\frac{1}{K}\left(P-P_0\right)\right)\hat{I}. \tag{3.6}$$

For small changes in moisture content and temperature, the equations (3.4)–(3.6) can be linearized by perturbations of these quantities:

$$S=\left(K\alpha_T+\frac{\partial P}{\partial T}\right)e+\rho_0 C_{W\varepsilon}\frac{\left(T-T_0\right)}{T_0};$$

$$\rho_0\mu=\left(K\beta_W+\frac{\partial P}{\partial W}\right)e-\rho_0\varepsilon_\Phi r^*. \tag{3.7}$$

To derive the heat conduction equations, it is possible to use the entropy balance equation [3]:

$$T\frac{\partial S}{\partial t}=-\vec{\nabla}\cdot j_q-\mu\frac{\partial\rho}{\partial t}+I, \tag{3.8}$$

where $\rho=\rho_0\left(1+W\right)$ is the density of the wet system; I is the total power of the heat sources; t is time; $\vec{j}_q=-\lambda_q\vec{\nabla}T$ is the heat flux.

Using the state equations, equation (3.8) is reduced to the heat conduction equation of the deformed system during drying:

$$\frac{\partial T}{\partial t}=a_q\nabla^2 T-\frac{K\beta_W e-\rho_0\varepsilon_\Phi r^*}{\rho_0 C_{W,\varepsilon}}\frac{\partial W}{\partial t}-\frac{e}{\rho_0 C_{W,\varepsilon}}\frac{\partial P}{\partial t}-T_0\frac{K\alpha_T+\alpha_p}{\rho_0 C_{W,\varepsilon}}\frac{\partial e}{\partial t}, \tag{3.9}$$

where $a_q=\dfrac{\lambda_q}{\rho_0 C_{W,\varepsilon}}$ is the thermal diffusivity coefficient; $\alpha_p=\dfrac{\partial P}{\partial T}$ is the temperature coefficient of excess water vapor pressure, such that $\frac{1}{\alpha_p}$ is the change in the phase transition temperature due to the emergence of excess pressure.

The linearized equation of diffusion-convective mass transfer, according to [11], is written as:

$$\frac{\partial W}{\partial t} = \vec{\nabla}\cdot\left(a_m\vec{\nabla}W + a_m\delta\vec{\nabla}T + \frac{K_p}{\rho_0}\vec{\nabla}P\right); \tag{3.10}$$

$$\frac{\partial P}{\partial t} = \vec{\nabla}\cdot\left(a_p\vec{\nabla}P\right) - \frac{\varepsilon_\Phi}{C_{W,\varepsilon}}\frac{\partial W}{\partial t}, \tag{3.11}$$

where a_m is the coefficient of molecular diffusion of substance absorption; a_p is the coefficient of convective diffusion; δ is the coefficient of thermogradient mass transfer; K_p is the permeability of the system.

The equations of motion for a deformable system, neglecting the convective velocity component, are given by:

$$\vec{\nabla}\cdot\hat{\sigma} + \vec{F} = \rho\frac{\partial^2\vec{u}}{\partial t^2}. \tag{3.12}$$

Here $\vec{F}$ is the external force vector per unit volume. The components of the stress tensor $\hat{\sigma}$ are determined by the relations (3.6). The relations (3.6), (3.7), (3.10)–(3.12) form a complete system and are used in the practice of investigating interconnected mechanical processes and heat-mass transfer processes. Considering the quasi-stationarity of the system's process, inertial forces in the equations of motion can be neglected. Then the equations of motion (3.12) can be written as:

$$\vec{\nabla}\cdot\hat{\sigma} + \vec{F} = 0. \tag{3.13}$$

When formulating problems of moisture-thermomechanics, it is necessary to add the appropriate boundary conditions to the equations (3.10)–(3.13). Initial conditions can be taken in the form:

$$T = T_0,\ W = W_0,\ P = P_0,\ \vec{u} = \vec{u}_0,\ \frac{\partial\vec{u}}{\partial t} = \vec{\upsilon}_0 \text{ at } t = t_0. \tag{3.14}$$

The boundary conditions for mechanical quantities involve specifying displacements or forces:

$$\vec{u} = \vec{u}_\Sigma \text{ or } \hat{\sigma}\cdot\vec{n} = \vec{\sigma}_\Sigma \text{ for } \vec{r}\in(\Sigma). \tag{3.15}$$

The surface conditions for thermal quantities can be of the first, second, or third kind, i.e.:

$$T = T_\Sigma \text{ or } \vec{q} \cdot \vec{n} = q_\Sigma \text{ or } \frac{\partial T}{\partial n} - \lambda_T \left(T - T_0\right) = 0, \tag{3.16}$$

where $\vec{n}$ is the normal to the surface (Σ). Similar conditions can be set for moisture content W:

$$W = W_\Sigma \text{ or } \vec{j}_w \cdot \vec{n} = j_\Sigma \text{ or } \frac{\partial W}{\partial n} - \lambda_W \left(W - W_0\right) = 0. \tag{3.17}$$

The boundary conditions for pressure P on the surface (Σ) typically reflect the equality of this pressure to the atmospheric pressure or the pressure in the boundary layer [5]:

$$P = P_\Sigma. \tag{3.18}$$

When considering semi-infinite regions, it is necessary to add appropriate conditions (boundedness of the solution) at infinity. For cylindrical or spherical regions using cylindrical or spherical coordinate systems, it is also necessary to formulate boundedness conditions on the cylinder axis or at the sphere center.

For isothermal processes, equation (3.9) is not considered, and equations (3.10), (3.11) are reduced to the system:

$$\frac{\partial W}{\partial t} = \vec{\nabla} \cdot \left(a_m \vec{\nabla} W + \frac{K_p}{\rho_0} \vec{\nabla} P \right);$$

$$\frac{\partial P}{\partial t} = \vec{\nabla} \cdot \left(a_p \vec{\nabla} P\right) - \frac{\varepsilon_\Phi}{C_{w,\varepsilon}} \frac{\partial W}{\partial t}. \tag{3.19}$$

This allows the moisture mechanics problem to be solved sequentially. In the first stage, it is necessary to find the solutions of the mass transfer problem (3.14), (3.17)–(3.19) with subsequent determination of the stress-strain state of the body from the mechanics equations (3.6), (3.13), (3.15). When solving the mechanics problem, the solutions of the mass transfer problem are used. Experimental studies show that the equilibrium distribution of liquid in the pore space during evaporation is determined by the relative vapor pressure $\chi = \frac{P}{P_s}$, or the related capillary pressure $P_k = \frac{RT}{V_L} \ln \chi$. As χ increases, the porous body is saturated with capillary condensate, and as χ decreases, evaporation occurs and the porous body is saturated with gas by moving the meniscus along the pore. This means that under nonequilibrium conditions, gradients of capillary and surface forces are the driving forces of mass transfer in the gas and liquid phases and have a decisive influence on the intensity of mass transfer. Gradients of partial and disjoining pressures arise due to the uneven moisture content during drying. During drying, the amount of liquid decreases due to the release of pores from condensate, thinning of films, and reduction of relative vapor pressure. Since the amount of liquid in films and in the gas phase is small compared to bulk liquid, it makes sense to only track changes in saturation. The space freed from the liquid is occupied by air. Since the molar volume of air is greater than the molar volume of the liquid, the air flow is much smaller than the liquid flow and can be neglected. The equations of motion during evaporation represent filtration equations complicated by diffusion in the gas phase.

3.1.3 MATHEMATICAL MODELING OF THE DRYING PROCESS OF POROUS BODIES CONSIDERING CAPILLARY PROPERTIES

To describe the mutual distribution of phases in a porous medium, structural models of porous media, particularly with irregular and random structures, are used. In this regard, it is necessary to use stochastic structural models [5]. The pore space is treated as a statistical ensemble of interconnected structural elements (pores), whose distribution is probabilistic. The most suitable model should be a simple statistical model that adequately describes the structure. To establish effective transfer coefficients for each phase, their connections as functions of macrovariables, conditions of the process in an individual pore, and geometric characteristics of the porous structure, the choice of an appropriate averaging technique is crucial. The necessary characteristics to describe the kinetics of the process in an individual pore are obtained based on mathematical models, the results of which are confirmed by experimental studies in capillaries [14].

The construction of capillary models is based on comparing known literature data on the sizes of the conductive elements of the material obtained through microscopic studies of the body's structure with the corresponding data obtained using capillary conductivity kinetics [5].

In real materials, whose pores represent a complex system of heterogeneous elements, capillary equilibrium is established, and the filling of an individual pore by a particular phase depends on the characteristics of the pore and its connections with other pores. The properties of the mutual distribution of phases are determined both by the distribution function of the structure characteristics of individual elements and by parameters reflecting the interconnection of these elements. The influence of the porous structure is taken into account by introducing effective coefficients of binary interaction into the Stefan-Maxwell equations [16]. Effective transfer coefficients are determined by the random geometry of the pore space, the nature of microinhomogeneities, or empirical dependencies that relate them to the parameters of the porous structure. In a quasi-homogeneous approximation, macroscopic transfer equations are applied, the structure of which coincides with the structure of equations valid in an individual elementary pore, in most cases in a straight cylindrical capillary.

The simplest model of the pore space of a polydisperse porous body is a system of cylindrical capillaries with ideal connections. In such a model, if the pores of radius r_f are filled with wetting liquid, then the pores of smaller radius are necessarily filled as well. This implies that each saturation value $\alpha_m = \bar{z}_m = \frac{L_m}{L}$ corresponds to a critical radius $r_f(\bar{z}_m)$ of the filled pores, where L_m is the moving phase transition coordinate. The walls of other pores with a radius $r > r_f$ are covered with equilibrium polymolecular films. The film flow equation in the porous medium can be represented as $j_{pl} = -\frac{\left[K_{pl}(\chi)\right]}{\mu_L V_L}\nabla P_L$, where $\left[K_{pl}(\chi)\right] = \Pi\int_{r_f}^{r_{\max}} \frac{h^3(r,\chi)}{r}\varphi(r)dr$ is the effective permeability coefficient of the films, which depends on the relative vapor pressure χ; $r_f(\chi)$ is the boundary radius of the filled pores at relative vapor pressure χ (in the gas zone $r_f(\chi) = r_{\min}$); Π is the porosity; μ_L is the viscosity.

The equilibrium film thickness h is determined by the relation $\frac{A}{6\pi h^3} = \frac{RT}{V_L}\ln\frac{P}{P_n}$. For water, $A = 7{\cdot}10^{-21}$ J; V_L is the molar volume; h is the equilibrium film thickness; R is the gas constant; T is the absolute temperature. The relative pressure of saturated vapor χ over the curved surface of the meniscus is determined by the Kelvin equation: $\chi = \frac{P}{P_n} = \exp\left\{-\frac{2\sigma_{Lg}\cos\theta V_L}{RTr}\right\}$, where P_n is the saturated vapor pressure over a free surface; σ_{Lg} is the surface tension; r is the radius. In the framework of the "well-mixed pores" model, the averaging operations for the gas phase are integration operations over r with a weighting function $\phi(r)$ within the limits $\left[r_f(r); r_{\max}\right]$, for example

$$K_{pl}(r) = \frac{2Ar_f(r)}{\mu\sigma_{Lg}}\int_{r_f(r)}^{r_{\max}} \frac{\varphi(r)dr}{2r - r_f(r)}.$$

Experimental studies show, and theoretical ones confirm, that polymolecular films should be taken into account only when $\chi > 0.96$. In capillaries with a radius of $r < 10$ nm, which are filled with capillary condensate at $\chi > 0.96$, film flow is not observed. In wide pores with molecular diffusion mode ($r > 500$–1000 nm), vapor flow prevails, while in narrow capillaries with a radius of $10 < r < 50$ nm, $\chi > 0.96$, film flow must be considered. In transitional pores ($50 < r < 500$ nm), the contributions of both mechanisms to mass transfer are comparable [3]. In pores with molecular diffusion mode, film flow can be neglected. In [3], it is argued that film transport of liquid due to the gradient of disjoining pressure and thermoosmosis at temperatures of the wet zone 60–70 °C can also be neglected. These assumptions are justified for intensive drying, where the heating of the wet zone occurs quickly, and the processes of internal evaporation and redistribution of film moisture have much less impact than the processes of moisture removal due to phase transitions.

The pressure in the gas zone consists of the partial pressures of vapor and air. In the case of a gas pore (a pore free of capillary condensate), the transfer is carried out both by convection and diffusion of the vapor-air mixture and by film flow under the influence of the disjoining pressure gradient.

The averaged flow in the gas zone is as follows:

$$\langle J(r)\rangle = \langle j_v\rangle + \langle j_{pL}\rangle + \langle j_D\rangle = -\rho_v \frac{K_g(r)}{\mu_g}\nabla P_g(r) + \rho_L \frac{K_{pL}(r)}{\mu_L}\nabla P_{kap}(r) + D_{1va}\nabla\rho_g,$$

where K_g and $K_{pL}(r)$ are the effective permeability coefficients for the gas zone and film filtration; D_{1va} and ρ_g are the effective diffusion coefficient and gas density, respectively, and $P_{kap}(r) = \frac{2\sigma_{Lg}\cos\theta}{r_f}$, where r_f is the critical radius. In the presence of a partial pressure gradient in the gas phase, vapor flow is observed in the capillary. The nature of gas movement in a straight cylindrical capillary is determined by the parameter $Kn = \frac{\lambda}{2r}$ (the Knudsen number, which represents the ratio of the number of molecular collisions with the walls to the number of intermolecular collisions), where λ is the mean free path length of the molecules, which depends on the gas composition and the total pressure of the mixture, and r is the radius. Depending on the Knudsen number, there are three characteristic regions of gas flow: Knudsen flow $(Kn \to \infty)$, transitional flow $(Kn = 1)$, and molecular flow $(Kn \to 0)$.

The use of the "dusty gas" model allows the identification of the structure of binary interaction coefficients in the Stefan-Maxwell system of equations. Within this model, interpolation formulas for effective diffusion coefficients in the transitional region between Knudsen and molecular diffusion are derived. They are also applied when the linear dimensions of the pores are so small that they are comparable to the mean free path length of the molecules. The generalized gas-kinetic binary diffusion coefficients, obtained in the first approximation of the Chapman-Enskog method, have the form:

$$D_{ij} = \left(\frac{3}{8}\pi RT / 2m_{ij}\right)^{\frac{1}{2}} \left(c\pi\sigma_{ij}^2\Omega_{ij}\right)^{-1},$$

where σ_{ij}^2 is the effective collision cross-section for the pair (i, j); $m_{ij} = \frac{M_i M_j}{(M_i + M_j)}$ is the reduced molecular mass; M_i is the molecular mass of the i-th component of the mixture; Ω_{ij} is the collision integral; R and T are the universal gas constant and the temperature of the mixture; $c = \sum c_i$; c_i is the number of moles of the i-th component per unit volume.

To describe the molecular flow of a binary gas mixture in a porous medium with micro- and macropores, the relation $j_i = D_{1ij}\frac{d\rho_i}{dz}$ is used. In this case, the effective diffusion coefficient can be taken as:

$$D_{1va} = D_{1av} = \left(1/D_{va}^{\infty} + 1/D_{va}\right)^{-1} = \left\{1/D^{\infty} + 1/D\ \right\}^{-1},$$

where D and D^{∞} are the molecular and Knudsen diffusion coefficients. At low Knudsen numbers, there is a viscous flow regime, in which the interaction of the gas with the capillary walls leads to slip with velocity υ_{si}. In this case, the average gas flow velocity is $\bar{\upsilon}_i = -\frac{r^2}{8\mu_i}\left(1 + 8\xi^* Kn_i\right)\frac{dP}{dz}$, where the second term expresses the slip effect; ξ^* is the dimensionless slip coefficient, which varies in the range of 0.67–1.43 [12].

The total convective flow of the gas mixture in the range from purely viscous flow to the transitional regime with $\xi^*_i = \frac{2}{3}$ is determined from the Weber equation $j_i = -\left\{\left(\frac{r^2}{8\mu}\right)P + D_i^{\infty}\frac{\pi/4 + Kn}{1 + Kn}\right\}\frac{d\rho}{dz}$. This interpolation formula provides accurate results for Knudsen, purely viscous, and transitional flows. The slip velocity is comparable to the average velocity even in capillaries whose radius exceeds the mean free path by tens of times. Only when $Kn \to 0$ can the slip effect be neglected. The mass flows J_i are related to the molar ratios $J_i = M_i j_i$.

At the phase transition boundary, the moisture density equals the critical density corresponding to the maximum negative pressures of the capillary moisture, which is in a bound state, causing moisture migration to the boundary. At this boundary, there is an energy jump equal to the amount of heat needed to convert the liquid to vapor. It should be noted that the phase transition temperature in a porous body also depends on the curvature of the interface. An analysis of stresses considering pore size dispersion and in the equivalent pore model [14] suggests that liquid evaporation occurs in a narrow zone separating areas occupied by liquid and gas.

A characteristic feature of describing the drying of capillary-porous bodies is the need to consider their capillary properties as fully as possible at the continuum level [5]. Note that the approach based on using the mass transfer potential [3] does not allow for this sufficiently.

In theoretical studies of heat and mass transfer processes, approaches are used where complex transfer processes in porous bodies are reduced to the Stefan problem with a moving phase boundary. In this approach, the evaporation conditions are recorded differently. The moisture content at the phase boundary can be taken as the initial moisture content, assuming that there is a moist area with the initial moisture content and a dry area where the moisture content is zero. In other studies, the moisture content at the phase boundary is taken as the average integral moisture content of the moist area located in the first half of the material's thickness behind the dry area.

In this regard, when describing the drying process, let's base it on the approach of [5], where the driving forces of mass transfer in the pores are taken to be the gradients of the partial pressures of the components of the pore gas and the capillary pressure. On this basis, it is possible to formulate a complete system of relations for the drying of porous bodies for characteristic methods of modeling external influences on them, neglecting the pore size dispersion. Let's separately consider the possibility of the drying process under the influence of an external constant electric field (electroosmotic drying).

3.1.4 CAPILLARY MODEL OF A POROUS BODY. PROBLEM STATEMENT

Let's consider a capillary-porous body occupying the region (V) of Euclidean space and bounded by a smooth surface (S). Let's assume that the porosity is open. At the initial moment of time, the body is saturated with liquid. Let's consider a class of materials whose transverse pore size is characterized by a small dispersion, which it is possible to neglect. The body on the surface (S) is in contact with a gas medium, which is a mixture of dry air and water vapor. Provided that the water vapor in the surrounding air relative to the body is unsaturated, the process of moisture evaporation from the pore space of the body and the gradual drying of the pores from the surface (S) into the body will occur. Due to the uniform transverse pore sizes, this process will proceed equally across the entire cross-section of the body, parallel to the surface (S). As a result, two zones will form – dried pores and liquid-filled pores, with the boundary of these zones in each pore being the meniscus of the liquid, the convexity or concavity of which is determined by the properties of the pore surface (its hydrophilicity or hydrophobicity). The surface enveloping the meniscus vertices will be denoted as (S^*).

Let's note that pore drying is never complete. In hydrophilic pores, water remains in the form of a wall layer (bound water). In the zone of dried pores, air and water vapor are present. It is possible to assume that the transverse pore size is significantly larger than the mean free path of the molecules present. This allows the expressions for the flows $\vec{j}_a$ and $\vec{j}_v$ of air and vapor in the dried zone to be written as [5]:

$$\vec{j}_k = \rho_k \vec{\upsilon} - D' \vec{\nabla} \rho_k,\ k = a, v,$$

where ρ_a and ρ_v are the densities of air and vapor, respectively; D' is an effective coefficient of binary diffusion in pores. The flow of air inside the body is quantitatively much smaller than the flow of steam outside, generated as a result of the water-steam phase transition. In this regard, in the future it is possible to neglect the air flow $\vec{j}_a$ putting $\vec{j}_a = 0$.

Let's also neglect the local change in vapor density ρ_v, assuming [5]:

$$\frac{\partial \rho_v}{\partial t} = 0,$$

which leads to the following vapor mass balance equation:

$$\vec{\nabla} \cdot \vec{j}_v = 0. \tag{3.20}$$

From these assumptions follow the isothermal Stefan-Maxwell equations for the binary gas mixture in the dried zone:

$$\rho_a\vec{\upsilon}-D'\vec{\nabla}\rho_a=0,\ \vec{\nabla}\cdot\left(\rho_v\vec{\upsilon}-D'\vec{\nabla}\rho_v\right)=0. \tag{3.21}$$

The average mass velocity $\vec{\upsilon}$ satisfies the Darcy equation [13]:

$$\vec{\upsilon}=-\frac{K_g}{\mu_g}\vec{\nabla}P_g. \tag{3.22}$$

It is possible to assume that the ideal gas law applies to the gas mixture $P_g=\left(\frac{\rho_a}{M_a}+\frac{\rho_v}{M_v}\right)RT$, where P_g is the gas pressure in the pores; M_a is the molar mass of air; M_v is the molar mass of vapor; R is the gas constant.

Using the state equation for the gas mixture and Darcy's law (3.22), it is possible to write the Stefan-Maxwell equations in terms of the key functions ρ_a and ρ_v (the densities of air and vapor):

$$\rho_a\frac{K_g}{\mu_g}\vec{\nabla}\left(\frac{\rho_a}{M_a}+\frac{\rho_v}{M_v}\right)RT+D'\vec{\nabla}\rho_a=0,$$

$$\vec{\nabla}\cdot\left[\rho_v\frac{K_g}{\mu_g}\vec{\nabla}\left(\frac{\rho_a}{M_a}+\frac{\rho_v}{M_v}\right)RT+D'\vec{\nabla}\rho_v\right]=0, \tag{3.23}$$

where K_g is the permeability coefficient dependent on the radius and shape of the pores; μ_g is the dynamic viscosity coefficient of the gas.

The written equations are valid in the region of dried pores, which is bounded by the surfaces (S) and (S^*). Note the non-linearity of the differential equation system (3.23).

On the moving surface (S^*), the vapor density can be taken as the saturated vapor density [5]:

$$\rho_v=\rho_{vn}. \tag{3.24}$$

The form of the boundary conditions on the surface (S) depends on the method of modeling the interaction of the body with the environment.

In the considered case of the body's surface (S) contacting with a medium that is a mixture of air and vapor, under natural drying conditions, it is possible to assume that at a distance from the surface (S), the densities ρ_{ve} of vapor and ρ_{ae} of air reach constant values ρ_{v1}, ρ_{a1}, respectively, which are characteristic of atmospheric air, i.e.:

$$\lim_{r\to\infty}\rho_{ve}=\rho_{v1},\ \lim_{r\to\infty}\rho_{ae}=\rho_{a1}, \tag{3.25}$$

where r is the distance from the surface (S).

In the region outside the body, mass transfer processes will be described by the Stefan-Maxwell equations under the condition of constant atmospheric pressure:

$$\vec{\nabla}\rho_{ae}-\frac{\rho_{ae}}{D'_e}\vec{v}_e=0,$$

$$\vec{\nabla}\cdot\left(\vec{\nabla}\rho_{ve}-\frac{\rho_{ve}}{D'_e}\vec{\upsilon}_e\right)=0, \qquad (3.26)$$

$$P_{ge}=\left(\frac{\rho_{ae}}{M_a}+\frac{\rho_{ve}}{M_v}\right)RT=const. \qquad (3.27)$$

On the surface (S), the normal component of the vapor flow from the drying body must be continuous, i.e.:

$$\vec{n}\cdot j_v=\left(\vec{\nabla}\rho_{ve}-\frac{\rho_{ve}}{D'_e}\vec{v}_e\right)\cdot\vec{n}, \qquad (3.28)$$

and the densities of the mixture components must be equal, which follows from the equality of the partial pressures on the surface (S):

$$\rho_{ae}=\rho_a,\;\; \rho_{ve}=\rho_v. \qquad (3.29)$$

Equations (3.23)–(3.29) constitute the complete system of relations and can be used to describe mass transfer during the natural drying of a porous body [5].

Often, when studying the drying process, especially under intensified blowing conditions, the external problem is posed only for the boundary layer of some finite thickness δ [5]. In this case, mass transfer in the boundary layer is described by the equations (3.23), (3.24). On the surface (S) of the body, the conjugation condition (3.29) holds, and on the outer surface (S^{**}) of the boundary layer, the condition:

$$\rho_{ae}=\rho_{a1}, \rho_{ve}=\rho_{v1}. \qquad (3.30)$$

It is possible to note that a number of authors [5] do not consider the boundary layer near the body surface, instead placing a Newton-type mass transfer condition on the surface (S):

$$\vec{n}\cdot j_v=\beta\left(\rho_v-\rho_{v1}\right), \qquad (3.31)$$

as well as the condition:

$$\rho_a=\rho_{a1}. \qquad (3.32)$$

Here β is the mass transfer coefficient of the body with the environment. In this case, the mass transfer parameter depends on the conditions of the impact (blowing) on the surface (S) of the body.

In this case, it is not possible to pose an external mass transfer problem and will describe the drying of the body with the system of relations (3.21)–(3.24).

To characterize the moisture content of the body, it is advisable to introduce the quantity:

$$\alpha_m = \frac{m_L}{m_{0L}}, \tag{3.33}$$

where m_L, m_{0L} are the current and initial masses of the liquid in the body, respectively. The rate of change of the liquid mass will be determined from the equation:

$$\frac{dm_L}{dt} = -\int_{(S)} \left(\vec{n} \cdot \vec{j}\right) dS, \tag{3.34}$$

under the initial condition:

$$m_L = m_{0L} \text{ at } t = 0. \tag{3.35}$$

The vector $\vec{j}_v$ of the vapor flux density from the body is the solution of the above-formulated problems (3.30); or (3.23), (3.24), (3.26); or (3.23), (3.35).

In terms of relative humidity, the equations (3.23), (3.35) have the form:

$$\frac{d\alpha_m}{dt} = -\frac{1}{m_{0L}} \int_{(S)} \left(\vec{n} \cdot \vec{j}\right) dS, \tag{3.35}$$

$$\alpha m = 1 \text{ at } t = 0. \tag{3.36}$$

The values of moisture content in the zones of dried and liquid-saturated pores are determined as follows:

$$W_v = \frac{\int_{(V_v)} \rho_v dV}{\rho_s V_s}; \tag{3.37}$$

in the zone of dried pores:

$$W_L = \frac{\rho_L V_L}{\rho_s V_s}, \tag{3.38}$$

in the zone of liquid-saturated pores, where (V_v) is the region of dried pores; V_v, V_L, V_s are the volumes of the regions of dried and liquid-filled pores and the volume of the skeleton.

It is possible to note that in general, due to the nonlinearity of the equations (3.23), (3.26), finding exact solutions to the drying problem constitutes a significant mathematical challenge. Approximate solutions to problems for canonical regions of a layer and a sphere are given, in particular, in the works [5].

Now proceed to Section 3.2 of our study, namely the mathematical model for analyzing the moisture and temperature distribution, radial displacements, and stresses within an individual particle of a multicomponent dispersed material of the capillary-porous structure during its convective drying.

3.2 MATHEMATICAL MODELING AND COMPUTATIONAL ANALYSIS OF MOISTURE, TEMPERATURE, AND STRESS DISTRIBUTION IN GRAIN IN CONVECTIVE DRYING. PROBLEM FORMULATION AND KEY SYSTEM OF EQUATIONS

Let's consider a layer of thickness L, which is blown by a drying agent (gas) with the convective transfer velocity in the intergranular medium for the falling-rate drying stage. The layer is composed in a certain way from identical wet grains, which we assume be of a R-radius spheres and is referenced to the Cartesian coordinate system so that the Oz axis is perpendicular to its surfaces.

The movement of gas in the stationary layer of dispersed particles is characterized by the flow around the particles that make up the layer and the flow in the channels between them. Each grain is considered a two-component solid solution (consisting of the main substance and moisture) of the so-called equivalent volume, which is referenced to the spherical coordinate system with the origin at its center ($r = 0$).

The drying process of the grain occurs through its outer surface of contact $r = R$ with the intergranular medium. The moisture concentration c_z at the location of the selected grain across the layer thickness z is determined from the solution of the mass transfer problem in the intergranular space, where the steam-air mixture is uniformly filtered in the complete displacement regime.

Under isothermal conditions, let's accept temperature T, the vector of radial displacements of the grain u_r, and moisture concentration c as the defining functions [3].

Based on the criterion of similarity of heat and mass transfer processes, the equations for the moisture concentration c_z and temperature T_z in the intergranular space of the layer are as follows [13]:

$$\frac{\partial c_z}{\partial \tau} + \upsilon \frac{\partial c_z}{\partial z} = D_z \frac{\partial^2 c_z}{\partial z^2} + J, \quad \frac{\partial T_z}{\partial \tau} + \upsilon \frac{\partial T_z}{\partial z} = \lambda_{Tz} \frac{\partial^2 T_z}{\partial z^2} + J_T, \tag{3.39}$$

where ∂T is the diffusion coefficient of moisture in the pore space; a_T is the thermal diffusivity; J, J_T are the intensities of the local sources of moisture and heat resulting from evaporation from individual grains; τ is time; z is the coordinate.

To determine the intensity of the local moisture source J, it is necessary to study the moisture diffusion in a separate grain.

It is possible to consider the grain substance as a two-component solid solution consisting of the main substance and moisture. The local thermodynamic state of such an isotropic system is determined by the values of the conjugate thermodynamic parameters: absolute temperature T, entropy S, components of the Cauchy stress tensor σ, strain ε, and chemical potentials of the solution components μ_k', concentrations $C_k (k = 1,2)$.

Then, applying the approaches of the theory of solid solutions [1] and choosing temperature T, displacement $\vec{u}$, and moisture concentration C_2 as the solving functions, under certain physical assumptions in the linear approximation, let's obtain:

1. Equation of state:

$$\sigma_{\alpha\beta} = \left[\left(K - \frac{2}{3}G\right)\varepsilon - K\left(\beta c + \alpha t\right)\right]\hat{I} + 2G\hat{\varepsilon}. \tag{3.40}$$

2. Equilibrium equations and compatibility conditions:

$$\vec{\nabla}\cdot\widehat{\sigma} = 0,\ Jnk\,\widehat{\varepsilon} = \vec{\nabla}\times\left(\vec{\nabla}\times\widehat{\varepsilon}\right)^{\mathrm{T}} = 0,$$

$$\widehat{\varepsilon} = \left[\vec{\nabla}\otimes\vec{u} + \left(\vec{\nabla}\otimes\vec{u}\right)^{\mathrm{T}}\right]\Big/2. \tag{3.41}$$

3. Balance equation of concentration and thermal conductivity equation:

$$\frac{\partial c}{\partial \tau} = D\Delta c - D_{\varepsilon}\Delta\varepsilon,$$

$$\vec{J} = -D\vec{\nabla}c + D_{\varepsilon}\vec{\nabla}\varepsilon,\ \frac{\partial t}{\partial \tau} = a_T\Delta t. \tag{3.42}$$

Here, K is the bulk modulus; G is the shear modulus; $\varepsilon = Tr\varepsilon$ is the first invariant of the strain tensor ε; β, α are the concentration and temperature coefficients of volumetric expansion; $c = C_2 - C_2^{(0)}$ and $t = T - T^{(0)}$ are deviations of moisture concentration and temperature from their values $C_2^{(0)}$ and $T^{(0)}$ in the initial state; $\hat{I}$ is the unit tensor; $\vec{\nabla}\cdot$ is the Hamiltonian operator; D is the diffusion coefficient; D_{ε} is the coefficient of the influence of the volumetric strain gradient on the mass flow; a_t is the thermal diffusivity coefficient; $\vec{J}$ is the moisture diffusion flux; $\Delta = \vec{\nabla}\cdot\vec{\nabla}$ is the Laplace operator; the symbols "$\cdot$", "Ч", "$\otimes$" denote the scalar, vector, and tensor products, respectively.

It should be noted that for the so-called "soft regimes" of drying, it is possible to assume that $(T - T_0)/T_0 << 1$.

A separate grain in the geometric approximation of a spherical particle is referenced to the spherical coordinate system with the origin at its center ($r = 0$). The drying process occurs through the surface $r = 0$ of contact with the intergranular air medium, whose characteristics depend on the coordinate z in the layer. Then, for the grain, the displacement vector has only a radial component u_r and accordingly:

$$\varepsilon_{rr} = \frac{du_r}{dr},\ \varepsilon_{\varphi\varphi} = \varepsilon_{\theta\theta} = \frac{u_r}{r},$$

$$\Delta c = \frac{1}{r}\frac{\partial^2 (rc)}{\partial r^2}, \vec{\nabla}\cdot\vec{\sigma} = \frac{\partial \sigma_{rr}}{\partial r} + \frac{1}{r}\left(2\sigma_{rr} - \sigma_{\theta\theta} - \sigma_{\phi\phi}\right). \tag{3.43}$$

From the equilibrium conditions (3.41), the state equations (3.40) [13], and the formulas (3.43), it is possible to obtain the relations of the volumetric strain gradients with the concentration expansion gradient.

From the equations (3.42) and (3.43), it is possible to obtain the key system of equations for a single sphere in the form of the displacement equation and the moisture concentration balance equation:

$$\frac{\partial}{\partial r}\left[\frac{1}{r^2}\frac{\partial\left(r^2 u_r\right)}{\partial r}\right] - \xi\left(\beta\frac{\partial c}{\partial r} + \alpha\frac{\partial t}{\partial r}\right) = 0,\quad \frac{\partial (rc)}{\partial \tau} = D\frac{\partial^2 (rc)}{\partial r^2} - \xi\beta D_\varepsilon \frac{\partial^2 (rc)}{\partial r^2}, \tag{3.44}$$

where $\xi = 3K/(3K + 4G)$ is the mechanical constant; $\varepsilon = \partial u_r/\partial r + 2u_r/r$.

At the initial moment of time ($\tau = 0$):

$$u_r = 0,\ c = c_0 = const. \tag{4.45}$$

On the surface of the sphere $r = R$, which is free from external loads, the condition $\hat{\sigma}\cdot\vec{n} = 0$ is written as:

$$\sigma_{rr} = \left(K + \frac{4}{3}G\right)\frac{\partial u_r}{\partial r} + 2\left(K - \frac{2}{3}G\right)\frac{u_r}{r} - K\left(\beta c + \alpha t\right) = 0. \tag{3.46}$$

It is possible to assume that the moisture flow on this surface is proportional to the concentration difference on the surface of the grain and the intergranular medium c_z: $c_R(\tau) \equiv c(R,\tau)$ [13]:

$$-D\frac{\partial c}{\partial r} + D_\varepsilon\frac{\partial \varepsilon}{\partial r} = k\left(c_R - c_z\right), \tag{3.47}$$

where k is the mass transfer constant on the surface of the sphere.

It is possible to assume that the sought functions and their derivatives at the center of the sphere (2.$r = 0$) are limited, in particular:

$$c(0,\tau) \neq \infty, \quad \left.\frac{\partial c}{\partial r}\right|_{r=0} = 0. \tag{3.48}$$

Since in the centrosymmetric problem, the relations hold:

$$\frac{\partial}{\partial r}\left[\frac{1}{r^2}\frac{\partial\left(r^2 u_r\right)}{\partial r}\right] = \frac{\partial \varepsilon}{\partial r}, \quad \frac{\partial \varepsilon}{\partial r} = \xi\left(\beta\frac{\partial c}{\partial r} + \alpha\frac{\partial t}{\partial r}\right), \tag{3.49}$$

the moisture balance equation (3.44) in the grain is written as:

$$\frac{\partial(rc)}{\partial \tau} = \tilde{D}\frac{\partial^2(rc)}{\partial r^2}, \tag{3.50}$$

and the boundary condition (3.47) is:

$$-\tilde{D}\left.\frac{\partial c}{\partial r}\right|_{r=R} = k\left(c_R - c_z\right) \text{ or } -\left.\frac{\partial c}{\partial r}\right|_{r=R} + H\left(c_z - c_R\right) = 0, \tag{3.51}$$

where $\tilde{D} = D - \xi\beta D_\varepsilon$ is the effective moisture diffusion coefficient; $H = k / \tilde{D}$ is the reduced mass transfer coefficient.

The heat conduction equation for temperature can be written as:

$$\frac{\partial(rt)}{\partial \tau} = a_T\frac{\partial^2(rt)}{\partial r^2}, \quad \left(0 < r < R\right). \tag{3.52}$$

The initial and boundary conditions for temperature are as follows:

$$-\left.\frac{\partial t}{\partial r}\right|_{r=R} + H_T\left(t_z - t_R\right) = 0, \quad \left.\frac{\partial t}{\partial r}\right|_{r=0} = 0,$$

$$t\left(0,\tau\right) \neq \infty, \; t\left(r,0\right) = f_T\left(r\right). \tag{3.53}$$

3.3 DEVELOPMENT OF SOLUTION ALGORITHMS FOR MOISTURE CONTENT AND STRESS-STRAIN STATE OF THE GRAIN

During the analysis of the moisture diffusion process in the grain, the moisture concentration in the intergranular space of the layer can be considered independent of time, i.e., $c_z = c_z(z)$. The function c_z is found from the solution of equation (3.39) under the corresponding boundary conditions in the quasi-stationary approximation (neglecting the partial derivative with respect to time, but the time dependence remains in the source). The solution of the problem is sought by the method of separation of variables [3]. Let's outline new functions:

$$\vartheta(r,z,\tau) = c_z(z) - c(r,z,\tau),\ \vartheta_T(r,z,\tau) = T_z(z) - T(r,z,\tau), \tag{2.54}$$

for which the problem (3.50)–(3.53) takes the form:

$$\frac{\partial\left(r\vartheta\right)}{\partial\tau} = \tilde{D}\frac{\partial^2\left(r\vartheta\right)}{\partial r^2},\ \frac{\partial\left(r\vartheta_T\right)}{\partial\tau} = \alpha_T\frac{\partial^2\left(r\vartheta_T\right)}{\partial r^2}, \tag{3.55}$$

$$\left.\frac{\partial\vartheta}{\partial r}\right|_{r=R} + H\vartheta_R = 0;\ \vartheta(0,\tau) \neq \infty,\ \left.\frac{\partial\vartheta}{\partial r}\right|_{r=0} = 0, \tag{3.56}$$

$$\vartheta(r,0) = c_{z0} - c_0,\ \left.\frac{\partial\vartheta_T}{\partial r}\right|_{r=R} + H_T\vartheta_{TR} = 0, \tag{3.57}$$

where $\vartheta_R(\tau) \equiv \vartheta\left(R,\tau\right)$, $\vartheta_{TR}(\tau) \equiv \vartheta_T\left(R,\tau\right)$.

Then the solution of the problem (3.55)–(3.57) is written as [1]:

$$\vartheta(r,z,\tau) = 2\delta c\sum_{n=1}^{\infty} A_n(\tau)\frac{\sin r_n}{r_n},\ \vartheta_T(r,z,\tau) = 2\delta T\sum_{n=1}^{\infty} A_{Tn}(\tau)\frac{\sin r_{Tn}}{r_{Tn}}, \tag{3.58}$$

where

$$\delta c = c_{z0}(z) - c_0,\ A_n(\tau) = \frac{\left(\sin\mu_n - \mu_n\cos\mu_n\right)e^{-\mu_n^2\tilde{D}\tau/R^2}}{\mu_n - \sin\mu_n\cos\mu_n},\ r_n = \mu_n r/R,$$

$$\delta T = t_{z0}(z) - t_0, A_{Tn}(\tau) = \frac{\left(\sin\mu_{Tn} - \mu_{Tn}\cos\mu_{Tn}\right)e^{-\mu_{Tn}^2\alpha_T\tau/R^2}}{\mu_{Tn} - \sin\mu_{Tn}\cos\mu_{Tn}},\ r_{Tn} = \mu_{Tn} r/R,$$

μ_n, μ_{Tn} are the roots of the characteristic equations:

$$\tan\mu = -\frac{\mu}{Fo-1}, \ \tan\mu_T = -\frac{\mu_T}{Fo_T-1}, \tag{3.59}$$

where $Fo = HR$, $Fo_T = H_T R$ are the Fourier numbers.

According to the relation (3.54), for the moisture concentration and temperature in the grain, let's find:

$$c(r,z,\tau) = c_z(z,\tau) - 2\delta c\sum_{n=1}^{\infty} A_n(\tau)\frac{\sin r_n}{r_n}, \ T(r,z,\tau) = T_z(z,\tau) - 2\delta T\sum_{n=1}^{\infty} A_{Tn}(\tau)\frac{\sin r_{Tn}}{r_{Tn}}. \tag{3.60}$$

For the moisture and temperature fluxes J_R, J_{TR} through the surface of the sphere:

$$J_R = -\tilde{D}\left.\frac{\partial c}{\partial r}\right|_{r=R} = -\frac{2\tilde{D}\left(\delta c\right)}{R}\sum_{n=1}^{\infty}\frac{A_n(\tau)}{\mu_n}\left(\sin\mu_n - \mu_n\cos\mu_n\right), \tag{3.61}$$

$$J_{TR} = -\lambda_T\left.\frac{\partial t}{\partial r}\right|_{r=R} = -\frac{2\lambda_T\left(\delta T\right)}{R}\sum_{n=1}^{\infty}\frac{A_{Tn}(\tau)}{\mu_{Tn}}\left(\sin\mu_{Tn} - \mu_{Tn}\cos\mu_{Tn}\right).$$

Then the intensity of the local sources of moisture and temperature in the case of evaporation from individual grains J, J_T, which appear in the original equation (3.39), will be:

$$J = \tilde{\alpha}(J_R + \delta_T J_{TR}), \ J_T = \tilde{\alpha}_T J_{TR}, \tag{3.62}$$

where $\tilde{\alpha}$, $\tilde{\alpha}_T$ are coefficients that depend on the size of the spheres (radius) and their packing (simple cubic, body-centered cubic, or face-centered cubic, etc.), δ_T is the thermogradient coefficient.

Given the accepted assumptions, the process of convective diffusion in the intergranular space of a layer of thickness $z = L$, composed of identical grains, is a fast process compared to the process of moisture diffusion from the volume of the grain to its surface. Therefore, further analysis of the drying process will be performed based on the equations:

$$D_z\frac{d^2c_z}{\partial z^2} - \upsilon\frac{dc_z}{\partial z} + J = 0, \lambda_{Tz}\frac{d^2t_z}{\partial z^2} - \upsilon\frac{dt_z}{\partial z} + J_T = 0, \tag{3.63}$$

where the function J is determined by expression (3.62). Considering that J, J_T can be written in the form (3.62), the equations (3.63) will be written as:

$$D_z \frac{d^2 c_z}{\partial z^2} - \upsilon \frac{dc_z}{\partial z} + c_z J_0(\tau) = c_0 J_0(\tau), \tag{3.64}$$

$$\lambda_{Tz} \frac{d^2 t_z}{\partial z^2} - \upsilon \frac{dt_z}{\partial z} + t_z J_{T0}(\tau) = t_0 J_{T0}(\tau).$$

Here,

$$J_0(\tau) = -\alpha \frac{2\tilde{D}}{R} \sum_{n=1}^{\infty} \frac{A_n(\tau)}{\mu_n} \left(\sin\mu_n - \mu_n \cos\mu_n \right),$$

$$J_{T0}(\tau) = -\alpha_{1T} \frac{2a_T}{R} \sum_{n=1}^{\infty} \frac{A_{Tn}(\tau)}{\mu_{Tn}} \left(\sin\mu_{Tn} - \mu_{Tn} \cos\mu_{Tn} \right).$$

Equations (3.64) will be solved under first-kind boundary conditions:

$$c_z\big|_{z=0} = 0, \ c_z\big|_{z=L} = c_z^n, \ t_z\big|_{z=0} = 0, \ t_z\big|_{z=L} = t_z^n;$$

and third-kind boundary conditions:

$$-D_z \frac{\partial c_z(0,\tau)}{\partial z} = \beta_z (c_z(0,\tau) - c_c), \ c_z\big|_{z=L} = c_z^n,$$

$$-\lambda_z \frac{\partial t_z(0,\tau)}{\partial z} = \beta_{Tz} (t_z(0,\tau) - t_c), \ t_z\big|_{z=L} = t_z^n, \tag{3.65}$$

where c_z^n is the concentration of saturated vapor, c_c is the concentration of the steam-air mixture in the external environment, t_z^n is the temperature corresponding to vapor saturation.

The solution of the problem (3.64), (3.65) is sought by the method of variation of constants [3]. After the corresponding calculations for the first case of boundary conditions, let's obtain:

$$c_z(z,\tau) = -K_1 e^{\lambda_1 z} + K_2 e^{\lambda_2 z} + c_0, \tag{3.66}$$

$$t_z(z,\tau) = -K_{T1} e^{\lambda_{T1} z} + K_{T2} e^{\lambda_{T2} z} + t_0,$$

where

$$K_1 = \left[c_0 e^{\lambda_2 L} + c_z^n - c_0 \right] / \Delta, \ K_2 = \left[c_0 e^{\lambda_1 L} + c_z^n - c_0 \right] / \Delta,$$

$$\Delta = e^{\lambda_2 L} - e^{\lambda_1 L};$$

$$K_{T1} = \left[t_0 e^{\lambda_{T2} L} + t_z^n - t_0\right] / \Delta_T, \; K_{T2} = \left[t_0 e^{\lambda_{T1} L} + t_z^n - t_0\right] / \Delta_T,$$

$$\Delta_T = e^{\lambda_{T2} L} - e^{\lambda_{T1} L};$$

$$\lambda_{1/2} = \upsilon \pm \sqrt{\upsilon^2 - 4D_z J_0(\tau)} / (2D_z),$$

$$\lambda_{T1/2} = \upsilon \pm \sqrt{\upsilon^2 - 4\alpha_{Tz} J_{T0}(\tau)} / (2\alpha_{Tz}).$$

Further, it is possible to use the dimensionless coordinate $\bar{z} = z / L$. Then:

$$c_z(\bar{z}, \tau) = -K_1 e^{\bar{\lambda}_1 \bar{z}} + K_2 e^{\bar{\lambda}_2 \bar{z}} + c_0, \tag{3.67}$$

$$t_z(\bar{z}, \tau) = -K_{T1} e^{\bar{\lambda}_{T1} \bar{z}} + K_{T2} e^{\bar{\lambda}_{T2} \bar{z}} + t_0,$$

where $\bar{\lambda}_i = L\lambda_i, \;\; \bar{\lambda}_{Ti} = L\lambda_{Ti}, \;\; i = 1,2.$

In the case of third kind boundary conditions, the sought functions c_z and t_z are also determined by formulas (3.67), in which:

$$K_1 = \left[\frac{\beta L}{D_z}(c_0 - c_c)e^{\bar{\lambda}_2} - (c_z^n - c_0)\left(\bar{\lambda}_2 + \frac{\beta L}{D_z}\right)\right] / \tilde{\Delta},$$

$$K_2 = \left[\frac{\beta L}{D_z}(c_0 - c_c)e^{\bar{\lambda}_1} + (c_z^n - c_0)\left(\bar{\lambda}_1 + \frac{\beta L}{D_z}\right)\right] / \tilde{\Delta},$$

$$\tilde{\Delta} = \left(\bar{\lambda}_1 + \frac{\beta L}{D_z}\right)e^{\bar{\lambda}_2} - \left(\bar{\lambda}_2 + \frac{\beta L}{D_z}\right)e^{\bar{\lambda}_1}.$$

Integrating the second equation (3.49), for the volumetric strain it is possible to obtain:

$$\varepsilon = \xi(\beta c + \alpha t) + L, \tag{3.68}$$

where the function $L(z,\tau)$, the "integration constant", is determined from the condition of zero strain at its center, i.e.:

$$L(z, \tau) = -\xi\left[\beta c(0, z, \tau) + \alpha t(0, z, \tau)\right].$$

From here:

$$\varepsilon(r,z,\tau)=\xi\left\{\beta\left[c(r,z,\tau)-c(0,z,\tau)\right]+\alpha\left[t(r,z,\tau)-t(0,z,\tau)\right]\right\},$$

and accordingly:

$$\varepsilon(r,z,\tau)=-2\xi\left\{\beta\delta c\sum_{n=1}^{\infty}A_n(\tau)\left(\frac{1}{r_n}\sin r_n-1\right)+\alpha\delta t\sum_{n=1}^{\infty}A_{Tn}(\tau)\left(\frac{1}{r_{Tn}}\sin r_{Tn}-1\right)\right\}. \tag{3.69}$$

The displacement u_r is found from the condition of zero radial stresses (3.46) on the surface of the sphere $r = R$. There is:

$$u_r(R,z,\tau)=-\frac{KR}{4G}\left\{\beta c(0,z,\tau)+\alpha t(0,z,\tau)\right\}. \tag{3.70}$$

To determine the radial displacement at an arbitrary point of the grain, it is possible to integrate the first expression (3.49). As a result, taking into account its limitation for $r = 0$ and the expression (3.70) for $r = R$, let's obtain:

$$u_r(r,z,\tau)=\frac{\xi\beta}{r^2}\left[c_z\frac{r^3}{3}-2R^3\delta c\sum_{n=1}^{\infty}\frac{A_n(\tau)}{\mu_n^3}\left(\sin r_n-r_n\cos r_n\right)\right]+\frac{r}{3}L_1(z,\tau)+$$

$$+\frac{\xi\alpha}{r^2}\left[t_z\frac{r^3}{3}-2R^3\delta t\sum_{n=1}^{\infty}\frac{A_{Tn}(\tau)}{\mu_{Tn}^3}\left(\sin r_{Tn}-r_{Tn}\cos r_{Tn}\right)\right]. \tag{3.71}$$

Here from formula (3.68):

$$L_1(z,\tau)=\left\{\frac{3K\beta}{4G}\left[-c_z+2\delta c\sum_{n=1}^{\infty}A_n(\tau)\right]-3\xi\beta\left[\frac{c_z}{3}-2\delta c\sum_{n=1}^{\infty}\frac{A_n(\tau)}{\mu_n^3}\left(\sin\mu_n-\mu_n\cos\mu_n\right)\right]+\right.$$

$$+\frac{3K\alpha}{4G}\left[-t_z+2\delta t\sum_{n=1}^{\infty}A_{Tn}(\tau)\right]-3\xi\alpha\left[\frac{t_z}{3}-2\delta t\sum_{n=1}^{\infty}\frac{A_{Tn}(\tau)}{\mu_{Tn}^3}\left(\sin\mu_{Tn}-\mu_{Tn}\cos\mu_{Tn}\right)\right].$$

In our case, the following components of the stress tensor are non-zero:

$$\sigma_{rr}=2G\frac{\partial u_r}{\partial r}+\left[\left(K-\frac{2}{3}G\right)\varepsilon-K\left(\beta c+\alpha t\right)\right], \tag{2.34}$$

$$\sigma_{\phi\phi}=\sigma_{\theta\theta}=2G\frac{u_r}{r}+\left[\left(K-\frac{2}{3}G\right)\varepsilon-K\left(\beta c+\alpha t\right)\right].$$

Based on the determined moisture concentration c and temperature t (formulas (3.66), (3.67)), displacement u_r (formula (3.71)), strain ε (formula (3.69)), let's obtain the stresses σ_{rr}, $\sigma_{\theta\theta}$, $\sigma_{\varphi\varphi}$ (formula (3.72)).

Thus, based on the proposed model, it is possible to calculate the stress-strain state of an individual grain, which depends on its location in the layer, the characteristics of the grain material and the intergranular space, the size and packing of the grains under different regimes of convective-diffusive moisture transfer.

CONCLUSIONS

Well-known mathematical models are typically based on quasi-homogeneous approximations. This approach is successfully applied to describe processes in which products and reactants form a single phase that uniformly fills the pore space. However, the application of interpenetrating continua methods is unjustified when studying processes with phase transformations and interphase boundaries. The interaction of phases determines capillary transport mechanisms. Capillary properties and phase transformation processes regulate the distribution of phases in the pore space and the magnitudes of interphase surfaces where heterogeneous transformations occur. Therefore, the use of capillary models, especially in the second stage of drying when phase transitions deepen, and the use of structural stochastic models are necessary.

Thus, the development of approaches and methods for mathematical modeling of heat and mass transfer processes in multiphase, multicomponent bodies, taking into account the sizes and nature of the phases, is relevant. In drying tasks, the efficiency of the process and the quality of the dried product are also important issues. This necessitates the study of the stress-strain state and the stability of the shape of thin-walled flat objects.

REFERENCES

1. Burak, Ya. Y., Chaplia, Ye. Ya., Chernukha, O. Yu. (2006). Kontynualno-termodynamichni modeli mekhaniky tverdykh rozchyniv. Kyiv: Naukova dumka, 272.
2. Sokolovskyy, Y., Levkovych, M., Mysyk, M. (2023). Matrix Approach to Numerical Modeling of Heat-and-Moisture Transfer Processes in a Medium with a Fractal Structure. 2023 17th International Conference on the Experience of Designing and Application of CAD Systems (CADSM), 44–48. https://doi.org/10.1109/cadsm58174.2023.10076519
3. Lykov, A. V. (1968). Teoriia sushki. Moscow: Energiia, 472.
4. Hachkevych, O. R., Kushnir, R. M., Terletskii, R. F. (2022). Mathematical Problems of Thermomechanics for Deformable Bodies Subjected to Thermal Irradiation. Ukrainian Mathematical Journal, 73 (10), 1522–1536. https://doi.org/10.1007/s11253-022-02011-7

5. Kheifitc, L. I., Neimark, A. V. (1982). Mnogofaznye protcessy v poristykh sredakh. Moscow: Khimiia, 320.
6. Pyanylo, Y. (2022). Analysis of Filtration Processes in Porous Environments Taking Into Aaccount the Movement of Capillaries. 2022 12th International Conference on Advanced Computer Information Technologies (ACIT), 9–12. https://doi.org/10.1109/acit54803.2022.9913087
7. Gayvas, B., Markovych, B., Dmytruk, A., Havran, M., Dmytruk, V. (2024). The methods of optimization and regulation of the convective drying process of materials in drying installations. Mathematical Modeling and Computing, 11 (2), 546–554. https://doi.org/10.23939/mmc2024.02.546
8. Hayvas, B., Dmytruk, V., Torskyy, A., Dmytruk, A. (2017). On methods of mathematical modeling of drying dispersed materials. Mathematical Modeling and Computing, 4 (2), 139–147. https://doi.org/10.23939/mmc2017.02.139
9. Baranovsky, S. V., Bomba, A. Ya. (2024). The diffusion scattering parameters identification for a modified model of viral infection in the conditions of logistic dynamics of immunological cells. Mathematical Modeling and Computing, 11 (1), 59–69. https://doi.org/10.23939/mmc2024.01.059
10. Mozhaev, A. P. (2001). Chaotic homogeneous porous media. 1. Theorems about structure. Engineering-Physical Journal, 74 (5), 196–201.
11. Lutcyk, P. P. (1988). Issledovanie protcessov teplomassoperenosa pri sushke kapilliarno-poristykh tel s uchetom vnutrennikh napriazhenii. Teplomassoobmen. Minsk: Institut teplo- i massoobmena im. A. V. Lykova, 184–197.
12. Khoroshun, L. P., Maslov, B. P., Leshchenko, P. V. (1989). Prognozirovanie effektivnykh svoistv pezoaktivnykh kompozitcionnykh materialov. Kyiv: Naukova dumka, 208.
13. Burak, Y., Kondrat, V., Gayvas, B. (2002). On the mathematical modeling of drying processes of porous bodies. Informational-Mathematical Modeling of Complex Systems. Lviv: CMM IPPMM named after Y. S. Pidstryhach NAS of Ukraine, Akhil Publishing House, 153–159.
14. Dibagar, N., Kowalski, S. J., Chayjan, R. A., Figiel, A. (2020). Accelerated convective drying of sunflower seeds by high-power ultrasound: Experimental assessment and optimization approach. Food and Bioproducts Processing, 123, 42–59. https://doi.org/10.1016/j.fbp.2020.05.014
15. Shlikhting, G. (1974). Teoriia pogranichnogo sloia. Moscow: Nauka, 690.
16. Gayvas, B. I., Dmytruk, V. A., Semerak, M. M., Rymar, T. I. (2021). Solving Stefan's linear problem for drying cylindrical timber under quasi-averaged formulation. Mathematical Modeling and Computing, 8 (2), 150–156. https://doi.org/10.23939/mmc2021.02.150
17. Gayvas, B. I., Dmytruk, V. A. (2022). Investigation of drying the porous wood of a cylindrical shape. Mathematical Modeling and Computing, 9 (2), 399–415. https://doi.org/10.23939/mmc2022.02.399
18. Gayvas, B. I., Markovych, B. M., Dmytruk, A. A., Havran, M. V., Dmytruk, V. A. (2023). Numerical modeling of heat and mass transfer processes in a capillary-porous body during

contact drying. Mathematical Modeling and Computing, 10 (2), 387–399. https://doi.org/10.23939/mmc2023.02.387

19. Gera, B., Kovalchuk, V., Dmytruk, V. (2022). Temperature field of metal structures of transport facilities with a thin protective coating. Mathematical Modeling and Computing, 9 (4), 950–958. https://doi.org/10.23939/mmc2022.04.950
20. Gayvas, B., Markovych, B., Dmytruk, A., Dmytruk, V., Kushka, B., Senkovych, O. (2023). Study of Contact Drying Granular Materials in Fluidized Bed Dryers. 2023 IEEE XXVIII International Seminar/Workshop on Direct and Inverse Problems of Electromagnetic and Acoustic Wave Theory (DIPED), 238–241. https://doi.org/10.1109/diped59408.2023.10269464
21. Alaa, K., Atounti, M., Zirhem, M. (2021). Image restoration and contrast enhancement based on a nonlinear reaction-diffusion mathematical model and divide & conquer technique. Mathematical Modeling and Computing, 8 (3), 549–559. https://doi.org/10.23939/mmc2021.03.549
22. Alaa, H., Alaa, N. E., Aqel, F., Lefraich, H. (2022). A new Lattice Boltzmann method for a Gray–Scott based model applied to image restoration and contrast enhancement. Mathematical Modeling and Computing, 9 (2), 187–202. https://doi.org/10.23939/mmc2022.02.187
23. Baala, Y., Agmour, I., Rachik, M. (2022). Optimal control of tritrophic reaction–diffusion system with a spatiotemporal model. Mathematical Modeling and Computing, 9 (3), 647–662. https://doi.org/10.23939/mmc2022.03.647
24. Bounkaicha, C., Allali, K., Tabit, Y., Danane, J. (2023). Global dynamic of spatio-temporal fractional order SEIR model. Mathematical Modeling and Computing, 10 (2), 299–310. https://doi.org/10.23939/mmc2023.02.299
25. Gouasnouane, O., Moussaid, N., Boujena, S., Kabli, K. (2022). A nonlinear fractional partial differential equation for image inpainting. Mathematical Modeling and Computing, 9 (3), 536–546. https://doi.org/10.23939/mmc2022.03.536
26. Najm, F., Yafia, R., Aziz Alaoui, M. A., Aghriche, A., Moussaoui, A. (2023). A survey on constructing Lyapunov functions for reaction-diffusion systems with delay and their application in biology. Mathematical Modeling and Computing, 10 (3), 965–975. https://doi.org/10.23939/mmc2023.03.965
27. Suganya, G., Senthamarai, R. (2022). Mathematical modeling and analysis of Phytoplankton–Zooplankton–Nanoparticle dynamics. Mathematical Modeling and Computing, 9 (2), 333–341. https://doi.org/10.23939/mmc2022.02.333
28. Ben-Loghfyry, A., Hakim, A. (2022). Time-fractional diffusion equation for signal and image smoothing. Mathematical Modeling and Computing, 9 (2), 351–364. https://doi.org/10.23939/mmc2022.02.351
29. Kostrobij, P. P., Markovych, B. M., Ryzha, I. A., Tokarchuk, M. V. (2021). Statistical theory of catalytic hydrogen oxidation processes. Basic equations. Mathematical Modeling and Computing, 8 (2), 267–281. https://doi.org/10.23939/mmc2021.02.267

30. Kostrobij, P. P., Ivashchyshyn, F. O., Markovych, B. M., Tokarchuk, M. V. (2021). Microscopic theory of the influence of dipole superparamagnetics (Type ⟨β -CD⟨FeSO4⟩⟩) on current flow in semiconductor layered structures (type gase, inse). Mathematical Modeling and Computing, 8 (1), 89–105. https://doi.org/10.23939/mmc2021.01.089
31. Aberqi, A., Elmassoudi, M., Hammoumi, M. (2021). Discrete solution for the nonlinear parabolic equations with diffusion terms in Museilak-spaces. Mathematical Modeling and Computing, 8 (4), 584–600. https://doi.org/10.23939/mmc2021.04.584
32. Bazirha, Z., Azrar, L. (2024). DDFV scheme for nonlinear parabolic reaction-diffusion problems on general meshes. Mathematical Modeling and Computing, 11 (1), 96–108. https://doi.org/10.23939/mmc2024.01.096
33. Baranovsky, S., Bomba, A., Lyashko, S., Pryshchepa, O. (2024). Diffusion Perturbations in Models of the Dynamics of Infectious Diseases Taking into Account the Concentrated Effects. Computational Methods and Mathematical Modeling in Cyberphysics and Engineering Applications 1, 273–303. https://doi.org/10.1002/9781394284344.ch11
34. El Hassani, A., Bettioui, B., Hattaf, K., Achtaich, N. (2024). Global dynamics of a diffusive SARS-CoV-2 model with antiviral treatment and fractional Laplacian operator. Mathematical Modeling and Computing, 11 (1), 319–332. https://doi.org/10.23939/mmc2024.01.319
35. Tokarchuk, M. V. (2023). Unification of kinetic and hydrodynamic approaches in the theory of dense gases and liquids far from equilibrium. Mathematical Modeling and Computing, 10 (2), 272–287. https://doi.org/10.23939/mmc2023.02.272
36. Belhachmi, Z., Mghazli, Z., Ouchtout, S. (2022). A coupled compressible two-phase flow with the biological dynamics modeling the anaerobic biodegradation process of waste in a landfill. Mathematical Modeling and Computing, 9 (3), 483–500. https://doi.org/10.23939/mmc2022.03.483
37. Pukach, P. Y., Chernukha, Y. A. (2024). Mathematical modeling of impurity diffusion process under given statistics of a point mass sources system. I. Mathematical Modeling and Computing, 11 (2), 385–393. https://doi.org/10.23939/mmc2024.02.385
38. Laham, M. F., Ibrahim, S. N. I. (2023). Penalty method for pricing American-style Asian option with jumps diffusion process. Mathematical Modeling and Computing, 10 (4), 1215–1221. https://doi.org/10.23939/mmc2023.04.1215
39. Dmytryshyn, L. I., Dmytryshyn, M. I., Olejnik, A. (2023). Model of money income diffusion in the European integration context. Mathematical Modeling and Computing, 10 (2), 583–592. https://doi.org/10.23939/mmc2023.02.583
40. Patil, J. V., Vaze, A. N., Sharma, L., Bachhav, A. (2021). Study of calcium profile in neuronal cells with respect to temperature and influx due to potential activity. Mathematical Modeling and Computing, 8 (2), 241–252. https://doi.org/10.23939/mmc2021.02.241