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CHAPTER 2

DRYING PROCESS MODELS FOR A MULTI-COMPONENT SYSTEM OF CAPILLARY-POROUS STRUCTURE BASED ON THERMODYNAMIC RELATIONSHIPS OF MIXTURE THEORY

ABSTRACT

In this Chapter, the main statements are formulated and fundamental thermodynamic relations for moisturized capillary-porous deformable systems are obtained when describing them using continuum representations. Possible methods of choosing the parameters of the local thermodynamic state of a solid deformable multi-component system are presented, being consistent with their choice of the liquid (gaseous) phase. A complete system of equations is constructed to describe the drying process of dense packing of capillary-porous materials, based on the approaches of the theory of the mixtures of porous and dense packing of disperse materials of multicomponent three-phase media.

There have been analysed the influence of the external heat flow, the initial volumetric moisture saturation on changes in temperature, volumetric moisture saturation, and air density in body pores in time by the example of conductive drying.

KEYWORDS

Mathematical modeling, continuum thermodynamics, drying, moisture, diffusion, capillary-porous, multi-component system, phase.

Recently, increased interest among scientists and researchers burst out for developing new and improving existing mathematical models and analytical-numerical methods for studying heat and mass transfer and the stress-strain state of porous materials, taking into account the influence of filtration, diffusion, and other physical processes due to the environmental situation on the planet. Problems of drying belong to such energy-consuming processes requiring new sustainable solutions.

To most accurately reproduce the physical content of the heat and moisture transfer processes in drying and adequately treat input data, such models are predominantly constructed based on general approaches and methods of thermodynamics of nonequilibrium processes. Integrating principles from thermodynamics, heat and mass transfer, and porous media mechanics, the model offers valuable insights and practical solutions for improving drying efficiency and effectiveness in various applications.

The application of heat and mass transfer models covers numerous engineering tasks, from agriculture and food processing to pharmaceuticals, construction, chemical engineering, environmental engineering, the energy sector, and the textile industry. These models are valuable for improving process efficiency, reducing energy consumption, and achieving sustainable development goals, making them reliable tools in modern engineering.

As a particular case of the heat and mass transfer model in an n-component, three-phase, deformable porous wet medium with phase transitions and chemical reactions, the mathematical model of drying capillary-porous bodies, examines the evolution of temperature, moisture content, pressure, kinematic characteristics of the process, and sensitivity to the influence of parameters and boundary conditions.

Mathematical models of drying of capillary-porous bodies are based on the laws of conservation of mass, momentum and energy, as well as known experimental dependencies, on the basis of which the equations of heat-mass-moisture transfer in the body are constructed. The ability to quantify the heat passing inside the body due to thermal conductivity is based on the Fourier hypothesis. At the same time, diffusion flows are taken into account on the basis of Fick's laws, filtration flows on the basis of Darcy's law.

Essential for advancing knowledge in science and engineering was a noticed similarity between heat and mass transfer processes and the universality of diffusion equations. By using common principles and mathematical descriptions, scientists and engineers develop more efficient and effective solutions for a wide range of applications.

Consider the results of state-of-the-art investigations in complex systems, which use models of heat and mass transfer.

B. Alaa et al. [1, 2] proposed a novel approach for image restoration and contrast enhancement using a nonlinear reaction-diffusion model. This model is based on the similarity of its behavior to a heat equation in low-gradient areas, while in high-gradient regions, diffusion is halted to preserve edges. The algorithm utilizes a divide-and-conquer technique coupled with a reaction-diffusion system. In [3], a new numerical approach is introduced using a Lattice Boltzmann method for a Gray-Scott based reaction-diffusion model aimed at image restoration and contrast enhancement. This method, traditionally used in fluid dynamics, effectively handles noisy images by comparing pixel motion to fluid motion.

V. Baala et al. [4] propose a new model of spatio-temporal dynamics concerning the tritrophic reaction-diffusion system, offering methodologies for managing optimal control of the system. G. Bounkaicha et al. [5] investigated spatio-temporal dynamics using a fractional order SEIR model, relevant for understanding drying processes. D. Gouasnouane et al. [6] developed a nonlinear fractional partial differential equation for image inpainting, applying nonlinear diffusive filters. M. Najm et al. [7] surveyed the construction of Lyapunov functions for reaction-diffusion systems with delay, providing stability analysis techniques necessary for drying process models. T. Suganya and S. Senthamarai [8] formulated a diffusive phytoplankton – zooplankton – nanoparticle model with a density-dependent death rate of predators, constructed it, and analyzed its local stability. D. Ben-Loghfyry and N. Hakim [9], based on a time-fractional diffusion equation, performed image and signal smoothing, offering an idea of using anomalous diffusion behaviors for modeling different phenomena of image processing.

A statistical description of catalytic hydrogen oxidation applied by Kostrobij [10] provides a comprehensive understanding of the complex interactions involved in catalytic reactions on metal surfaces, incorporating both the diffusion of reactants and the magnetic properties of the ions and atoms involved. A generalized Cattaneo-type diffusion equation in time fractional derivatives is obtained in [11] for electrons with a characteristic relaxation time, and a generalized model is proposed based on a statistical approach that accounts for the complexity of relaxation electromagnetic diffusion processes for electrons in layered nanostructures. T. Aberqi et al. [12] provided a discrete solution for nonlinear parabolic equations with diffusion terms. They proved the existence and uniqueness of a weak solution using an approximation approach combining internal approximation with the backward Euler scheme, and provided a priori error estimates for temporal semi-discretization. F. Bazirha and S. Azrar [13] developed a DDFV scheme for nonlinear parabolic reaction-diffusion problems on general meshes, applicable to complex geometries in capillary-porous structures. B. Gayvas et al. [14] addressed solving Stefan’s linear problem for drying cylindrical timber, offering solutions for phase change problems in drying.

Important practical problems in medicine are proposed in [15, 16]. D. Baranovsky and T. Bomba identified diffusion scattering parameters for a modified model of viral infection [15]. M. El Hassani et al. examined the dynamics of a diffusive SARS-CoV-2 model using fractional Laplacian operators [16].

Consideration of a broader range of conditions and parameters that affect heat and mass transfer processes is realized in [17], where S. Tokarchuk has unified kinetic and hydrodynamic approaches in the theory of dense gases and liquids far from equilibrium, under arbitrary Knudsen number conditions. The collision integral of this equation includes the diffusion coefficient in velocity space. Insights into liquid and gas interactions are provided by L. Belhachmi et al. [18], who discussed coupled compressible two-phase flow.

E. Pukach and T. Chernukha [19] focused on impurity diffusion processes, essential for accurately describing mass transfer equations. O. Ogunmiloro et al. [20] focused on fractional order spatial models, highlighting computational analysis to ensure the existence and uniqueness of solutions, which is critical for the reliability of drying process models.

A. Dmytryshyn et al. [21] modeled the diffusion of money income, providing methodologies for solving this problem. D. Laham and H. Ibrahim [22] proposed a penalty approach for pricing the American-style Asian option under the Merton model, which is particularly relevant for today’s global financial markets. By including jump-diffusion in the models, Laham’s approach captures the skewness and kurtosis features of return distributions often observed in several assets.

S. Pradhan et al. [23] modeled mixed-traffic in urban areas, where advection equation captures the bulk movement of cars, while the advection-diffusion equation includes the effects of diffusion, providing a more detailed description of the motorbike flow. In [24], B. Gayvas et al. presented an approach to optimizing the convective drying process by leveraging empirical relationships and drying technology principles through improved accounting for thermal diffusion.

Models of capillary-porous materials require researchers to carefully study the effects on the transfer processes of capillary forces and diffusion processes, so let's proceed to the theory.

The drying process is characterized by changes in temperature, volume, and composition of the system. Let's assume that the change in composition is possible only due to the phase transformation of liquid into vapor and vapor into liquid, and is determined by the change in the density of the components $\rho_i = \lim\limits_{\Delta V \to 0} \frac{\Delta m_i}{\Delta V}$, $(i = 0, L, v, a)$. No chemical reactions are involved. Let S_i be the partial entropy; ε_{ij}, σ_{ij}^0 are the strain and stress tensors of the solid skeleton; P_i is the pressure tensor of the i-th component; T_i is the temperature; ρ_i is the density; μ_i is the chemical potential. All the functions depend on the parameters ε_{ij}, T_i, ρ_i, P_i, with $\varepsilon_{ij} = 0$, $T = T_0$, $\rho = \rho_{i0}$ in the initial state.

Let's elaborate on describing the densities of the porous (granular) medium $\rho_i = \frac{M_i}{V}$, where M_i is the mass of the system components, $M = \sum\limits_i M_i$, $V = V_0 + V_H$, and V_H is the volume of the carrier (continuous phase) $V_H = V_a + V_L + V_v$. $\upsilon_H = \frac{1}{\rho_a} + \frac{1}{\rho_L} + \frac{1}{\rho_V}$ is the specific volume of the carrier phase. Let $\Pi = \frac{V_H}{V_H + V_0}$ be the porosity of the medium and V_0 the volume of the solid phase. Also, let's introduce the true densities of the components $\rho_i^0 = \frac{M_i}{V_i}$. To derive the main equations describing the transfer processes in a thick layer, the method of local volume averaging will be employed. According to this method, each point of the porous medium is mapped to the small volume V, bounded by the closed surface S. There are two types of parameter averaging in the main equations: averaging values of local volume and averaging phase values (true). The averaging volume values Φ for a phase i are defined as follows: $\langle \Phi \rangle = \frac{1}{V} \int\limits_{V_i} \Phi dV$, and the averaging phase ones are defined as $\langle \Phi \rangle^i = \frac{1}{V_i} \int\limits_{V_i} \Phi dV$, where V_i is the volume occupied by the i-th phase. Assuming that the carrier phase is a mixture of liquid and gas, with the latter being a homogeneous mixture of an ideal gas – air and vapor, the following is obtained:

$$\rho_a=\frac{M_a}{V}=\frac{M_a}{V_a}\frac{V_a}{V_H}\frac{V_H}{V_H+V_0},\quad \rho_v=\frac{M_v}{V}=\frac{M_v}{V_v}\frac{V_v}{V_H}\frac{V_H}{V_H+V_0},$$

$$\rho_L=\frac{M_L}{V}=\frac{M_L}{V_L}\frac{V_L}{V_H}\frac{V_H}{V_H+V_0},\quad V_H=V_L+V_g.$$

Let’s introduce the volume saturation with liquid $\alpha=\frac{V_L}{V_H}$, $1-\alpha=\frac{V_g}{V_H}$, and considering that $V_v=V_a=V_g$, the expressions for the scaled densities in terms of the true ρ_i^0 can be obtained in the following form [25]:

$$\rho_0=\left(1-\Pi\right)\rho_0^0=\alpha_1\rho_0^0,\quad \rho_L=\Pi\alpha\rho_L^0=\alpha_L\rho_L^0,\quad \rho_v=\Pi\left(1-\alpha\right)\rho_v^0=\alpha_v\rho_v^0,$$

$$\rho_a=\Pi\left(1-\alpha\right)\rho_a^0=\alpha_a\rho_a^0,\quad \rho_g=\rho_a+\rho_v=\Pi\left(1-\alpha\right)(\rho_a^0+\rho_v^0)=\alpha_g(\rho_a^0+\rho_v^0).$$

During evaporation, the volume saturation α changes. Assuming that drying loss is possible during the drying process, the volume of the skeleton decreases, with the positions of the skeleton particles in space undergoing change. The drying-up can be characterized by a changing volume concentration Π. At constant temperature and volume, the chemical potential μ_β according to the formulas for internal energy $du_\beta=T_\beta dS_\beta+\mu_\beta d\rho_\beta-P_\beta dV_\beta\left(\beta=a,v,\ L\right)$, $du_0=TdS+\mu_0 d\rho_0-\sigma_{ij}d\varepsilon_{ij}$ and heat of phase transition, can be given in the form $\mu_\beta\Big|_{T,V_\beta}=\frac{\partial u_\beta}{\partial\rho_\beta}-\frac{1}{2}\sum_{\gamma=1}^{\beta}r_{\beta\gamma}\frac{\partial\rho_{\beta\gamma}}{\partial\rho_\beta}$, where $\partial\rho_{\beta\gamma}$ is the change in density of a component β due to phase or chemical transformation of the component γ, $r_{\beta\gamma}$ being the specific heat of phase transition or chemical transformation of the component γ in the component β. The derivative $\frac{\partial u_\beta}{\partial\rho_\beta}$ determines the change in the specific internal energy caused by the change in the mass of the component β due to phase and chemical transformations. To express the free energy of an elementary volume, let the free energy components be averaged by phase. Moving from phase averaging to volume averaging, the ratio $\left\langle f\right\rangle_V=\alpha_i\left\langle f\right\rangle_i$ is taken into account. Then the brackets denoting volume averaging are omitted. Provided that there is no deformation ε_{ij} and no change in volume V, the free energy function of the i-th phase can be expressed as follows:

$$F_i=\int_{T_0}^{T}\left(\frac{\partial F_i}{\partial T_i}\right)_{\rho,\varepsilon=0}dT_i+\int_{\rho_{i0}}^{\rho_i}\left[\frac{\partial u_i}{\partial\rho_i}-\frac{1}{2}\sum_{\gamma=1}^{i}r_{i\gamma}\frac{\partial\rho_{i\gamma}}{\partial\rho_i}\right]d\rho_i. \tag{2.1}$$

The free energy function of an elementary volume is of the form:

$$F^0=\alpha_0\int\limits_{T_0}^{T}\left(\frac{\partial F_0}{\partial T_0}\right)_{\rho,\varepsilon=0}dT_0+\alpha_i\sum_i\int\limits_{T_0}^{T}\left(\frac{\partial F_i}{\partial T_i}\right)_{\rho,\varepsilon=0}dT_i+\sum_\beta\alpha_\beta\int\limits_{\rho_{\beta0}}^{\rho_{\beta i}}\left[\frac{\partial u_\beta}{\partial\rho_\beta}-\frac{1}{2}\sum_{\gamma=1}r_{\beta\gamma}\frac{\partial\rho_{\beta\gamma}}{\partial\rho_\beta}\right]d\rho_\beta.$$

Let's assume a single-phase model for heat transfer ($T = T_i$, $dT = dT_i$) and introduce the scaled heat capacity of wet material. To this end, the entropy dependence on the temperature at constant volume and concentration is expressed in the form $S_{\varepsilon_{ij},\rho_\beta}=\int\limits_{T_0}^{T}\rho_0(C_{ef}/T)dT$. In expressions (2.1), the derivatives $\left(\frac{\partial F_i}{\partial T}\right)_{\rho,\varepsilon=0}$ are defined through the heat capacities of the C_i system components in the isochoric process $\rho_i C_i\big|_{\rho,\varepsilon}=T_i\left(\frac{dS_i}{dT_i}\right)_{\rho,\varepsilon}=-T_i\frac{d^2F_i}{dT_i^2}{}_{\rho,\varepsilon}$ $(i=0,L,v,a)$, which follows that $\left(\frac{\partial F_i}{\partial T}\right)_{\rho,\varepsilon}=-\int\limits_{T}^{T_i}\frac{\rho_i^0C_i}{T_i}dT_i$. The expression for the effective heat capacity in terms of the heat capacities of the components is of the form:

$$C_{ef}=C_0+\frac{\alpha_L\rho_L^0C_L+\alpha_v\rho_v^0C_v+\alpha_a\rho_a^0C_a}{\rho_0}=C_0+\frac{\rho_LC_L+\rho_vC_v+\rho_aC_a}{\rho_0}.$$

The explicit form of the functions S, σ_{ij}, μ_i can be found by expanding the function $F=F_0+\sum_i F_i$ in a Taylor series in powers ε_{ij} through the first two invariants $I_1=\varepsilon_1=\varepsilon_{kk}$, $I_2=\varepsilon_2=\varepsilon_{ij}$ of this tensor ($k = i = j = 1, 2, 3$) and by retaining terms F^0 in the expansion no higher than the second order of smallness [26]. As in the previous paragraph, the effective stresses on the elementary site $\sigma_{ij}=\sigma_{ij}^c(1-\Pi)+\sigma_{ij}^H\Pi=\sigma_{ijc}+\sigma_{ijH}$.

The application of surface forces σ_{ijH} leads to the movement of the load-bearing phase and system deformation. The determination of the stress tensor σ_{ijH} is related to solving the problem of the flow of a continuous phase in the system.

The volume-averaged free energy of elastic deformation can be determined by analogy with a continuous medium. Then the total free energy is expressed as follows [29]:

$$F(\varepsilon_{ij},T,w)=1/2(K-2/3G)\varepsilon_{kk}^2+$$

$$+1/3K\left[\alpha_{t0}(T-T_0)+\sum_{\beta=0,L,v,a}\alpha_{\beta0}(\rho_\beta-\rho_{\beta0})(2/3G+\lambda)-\alpha_pP_H\right]\varepsilon_{kk}+$$

$$+G\varepsilon_{ij}\varepsilon_{ij}+\sum_{\beta=0,L,v,a}\alpha_{\beta}\int_{\rho_{\beta 0}}^{\rho_{\beta i}}\left[\frac{\partial u_{\beta}}{\partial\rho_{\beta}}-\frac{1}{2}\sum_{\gamma=1}r_{\beta\gamma}\frac{\partial\rho_{\beta\gamma}}{\partial\rho_{\beta}}\right]d\rho_{\beta}-\int_{T_0}^{T}dT\int_{T_0}^{T}\frac{\rho_0 C_{ef,w,\varepsilon}}{T}dT, \qquad (2.2)$$

where $\frac{\partial F}{\partial I_1}=-\frac{K}{3}\varepsilon_{kk}$, $\frac{\partial F}{\partial I_2}=G=\frac{E}{2(1+\nu)}$, $\frac{\partial^2 F}{\partial I_1^2}=(K-2/3G)=\lambda=\frac{E\nu}{(1+\nu)(1-2\nu)}$,

with Poisson's ratio ν; Laham's ratio λ, Young's modulus $E=\frac{(1+\nu)(1-2\nu)}{\nu}(K-2/3G)$; the bulk elasticity modulus K; the shear modulus G; the resultant change in the unit volume of the body in the absence of stresses ε; the initial temperature T_0; the current temperature T; the pore pressure P; the atmospheric pressure P_0; $P_H = P - P_0$; the linear thermal expansion coefficient α_{Tw}; the linear shrinkage coefficient β_{Tw}.

The relationship between the average stress tensor, which determines the contribution to the macrodeformation of a granular heterogeneous system under drying conditions, is given as follows: $\sigma_{ij}=\left(\sigma_{ij}^{f}-P_H\delta^{ij}\right)$, where σ_{ij}^{f} is the effective stress tensor and is expressed by Hooke's law through the solid phase strain tensor as follows:

$$\sigma_{ij}^{f}=(1-\Pi)\Big\{\lambda_f^{\sigma}\varepsilon_{mm}\delta^{ij}+2\mu_f^{\sigma}\varepsilon_{ij}^{0}+\nu_f^{\sigma}P\delta^{ij}-\nu_f^{\sigma}\left[\lambda_0^{\sigma}+2/3\mu_0^{\sigma}\right]\times$$

$$\times\left[\alpha_{t0}\left(T-T_0\right)+\sum_{\beta=0,L,v,a}\alpha_{\beta 0}\left(\rho_{\beta}-\rho_{\beta 0}\right)\right]\delta^{ij}\Big\}.$$

Here, λ_f^{σ}, μ_f^{σ}, ν_f^{σ} are the effective coefficients, the volume fraction of the skeleton in the system being 1-Π. As regards a capillary-porous elastic-plastic body, $(1-\Pi)\mu_f^{\sigma}(1-\omega)=\mu(1-\omega)=G$ is the effective strain shear modulus, with $\omega=\omega(\varepsilon,T,\rho_i)$ as a function of relative shear in the plastic strain region determined from measurements of generalized strains $\varepsilon=\frac{\sqrt{2}}{2(1+\nu)}\left[\left(\varepsilon_{ij}-\varepsilon_{ii}\right)^2+6\varepsilon_{ij}^2\right]^{0,5}$ $(i\neq j)$, ν as Poisson's ratio and generalized stresses for simple loading cases; $(1-\Pi)\lambda_f^{\sigma}=\lambda$ is the generalized Laham's constant. As far as a granular medium is concerned, $(1-\Pi)\mu_f^{\sigma}=(1-\Pi)\mu^{\sigma}\left(1+\frac{\mu^{\sigma}}{\mu_0^{\sigma}}\right)^{-1}$; $(1-\Pi)\lambda_f^{\sigma}=(1-\Pi)\lambda^{\sigma}\left(1+0\left(\frac{\lambda^{\sigma}}{\lambda_0^{\sigma}},\frac{\mu^{\sigma}}{\mu_0^{\sigma}}\right)\right)^{-1}$; $(1-\Pi)\lambda_f^{\sigma}$, $(1-\Pi)\mu_f^{\sigma}$ are the elastic moduli of the granular skeleton depend on the structure and bonds between the grains (the greater the porosity, the lower the elastic moduli, other parameters being equal). Here, λ_0^{σ}, μ_0^{σ} are Laham's constants of an elastic grain;

$(1-\Pi)\left(\lambda_f^\sigma+2/3\mu_f^\sigma\right)=\lambda+2/3\mu$, $\mu=\dfrac{E}{2(1+\nu)}$, E is the effective Young's modulus. For a granular medium, the effective coefficients of thermal expansion and shrinkage, expressed through the effective elastic moduli, are as follows:

$$\alpha_t=(1-\Pi)\nu_f^\sigma\left(\lambda_0^\sigma+2/3\mu_0^\sigma\right)\alpha_{t0};\ \ \alpha_\beta=(1-\Pi)\nu_f^\sigma\left(\lambda_0^\sigma+2/3\mu_0^\sigma\right)\alpha_{\beta 0},$$

$$\nu_f^\sigma=\frac{\lambda^\sigma+2/3\mu^\sigma}{\left(\lambda_0^\sigma+2/3\mu_0^\sigma\right)}\left[1+0\left(\frac{\lambda^\sigma}{\lambda_0^\sigma},\ \frac{\mu^\sigma}{\mu_0^\sigma}\right)\right],\ \ K=(1-\Pi)\left(\lambda^\sigma+2/3\mu^\sigma\right),$$

are the elastic deformation coefficients averaged over the elementary volume. For a wet porous material, all these values are determined on the basis of experimental studies, and as practice shows, they are functions of moisture content and temperature. Based on the above, the expressions of entropy and chemical potential have the following form:

$$S=-\varepsilon_{ij}\varepsilon_{ij}\frac{\partial\mu}{\partial T}-\varepsilon_{kk}^2\frac{\partial\lambda}{\partial T}+K\varepsilon_{kk}\frac{\partial}{\partial T}\left[\alpha_t\left(T-T_0\right)+\sum_\beta\alpha_\beta\left(\rho_\beta-\rho_{\beta 0}\right)\right]-\frac{\partial\alpha_1}{\partial T}\frac{P_1}{E_0}+\int_{T_0}^{T}\frac{\rho_0 C_{ef}}{T}dT,\quad(2.3)$$

$$\rho_0\mu_\beta=\varepsilon_{ij}\varepsilon_{ij}\frac{\partial\mu}{\partial\rho_\beta}+\varepsilon_{kk}^2/2\frac{\partial\lambda}{\partial\rho_\beta}-K\varepsilon_{kk}\frac{\partial}{\partial\rho_\beta}\left[\alpha_t\left(T-T_0\right)+\sum_\beta\alpha_\beta\left(\rho_\beta-\rho_{\beta 0}\right)\right]-$$
$$-\frac{\partial u_\beta}{\partial\rho_\beta}-\frac{1}{2}\sum_\gamma r_{\beta\gamma}\frac{\partial\rho_{\beta\gamma}}{\partial\rho_\beta}+E_0\varepsilon_{kk}\frac{\partial\left(\alpha_1 P_1\right)}{\partial\rho_\beta}.\quad(2.4)$$

2.1 ENTROPY BALANCE EQUATION

According to the basic principles of the thermodynamics of irreversible processes, the product of absolute temperature and entropy growth rate is equal to the sum of the products of fluxes and the corresponding thermodynamic forces. Then, the energy transfer equations can be expressed in the form [30]:

$$\rho\frac{\partial S}{\partial\tau}=\vec{\nabla}\cdot\left(\lambda_q\vec{\nabla}T\right)+\vec{\nabla}\cdot\left(\lambda_{t\beta}\vec{\nabla}\rho_\beta\right)+\vec{\nabla}\cdot\left(\lambda_{tp}\vec{\nabla}P\right)+\rho_\beta\mu_\beta\frac{\partial\rho_\beta}{\partial\tau}+\Pi_t.\quad(2.5)$$

Here, λ_q, $\lambda_{t\beta}$, λ_{tp} are the coefficients of heat transfer; Π_t is the part of entropy production related to the redistribution of heat and mass in the body volume; the quantity of $\rho_\beta\mu_\beta\dfrac{\partial\rho_\beta}{\partial\tau}$, resulting from the part of entropy production due to evaporation.

For small changes in temperature, and mass content, at which the characteristics of the medium can be considered constant, equations (2.3), (2.4) have the form:

$$S = K\alpha_T\varepsilon_{kk} - \frac{\partial}{\partial T}\left(\alpha_1 P_1\right) + \frac{\rho_0 C_{ef}}{T_0}\left(T - T_0\right);$$

$$\rho_0\mu_\beta = -K\alpha_\beta\varepsilon_{kk} + \frac{\partial}{\partial\rho_\beta}\left(\alpha_1 P_1\right) - \varepsilon_{\Phi\beta}r^*,\ \varepsilon_{\Phi 0} = \varepsilon_{\Phi a} = 0, \tag{2.6}$$

where $r^* = 1/2r_{Lv}$, $\varepsilon_{\Phi L} = \dfrac{\partial\rho_{Lv}}{\partial\rho_L}$ is the degree of phase transition completeness; r^* is heat of the phase transition. In equations (2.6), it is assumed that $\ln\dfrac{T}{T_0} = \ln\left(1 + \dfrac{\theta}{T_0}\right) \approx \dfrac{T - T_0}{T_0}$. Let's consider $\alpha_1 P_1 = P$, $\lambda_{tp} = 0$, $\lambda_{t\beta} = 0$. The entropy balance equation in this case is expressed as follows:

$$T\frac{d}{d\tau}\left[K\alpha_T\varepsilon_{kk} + \frac{\partial P}{\partial T}\varepsilon_{kk} + T\frac{\rho_0 C_{ef}}{T_0}\right] = \lambda_q\nabla^2 T - K\alpha_{L0}\varepsilon_{kk}\frac{d\rho_L}{d\tau} - K\alpha_{v0}\varepsilon_{kk}\frac{d\rho_v}{d\tau} - K\alpha_{a0}\varepsilon_{kk}\frac{d\rho_a}{d\tau} -$$

$$-K\alpha_{00}\varepsilon_{kk}\frac{d\rho_0}{d\tau} - \varepsilon_{kk}\frac{dP}{d\rho_L}\frac{d\rho_L}{d\tau} - \varepsilon_{kk}\frac{dP}{d\rho_v}\frac{d\rho_v}{d\tau} - \varepsilon_{kk}\frac{dP}{d\rho_a}\frac{d\rho_a}{d\tau} - \varepsilon_\Phi r\frac{d\rho_v}{d\tau} + I_g, \tag{2.7}$$

where α_{i0} are the initial volume concentrations. Here, all the quantities included in equation (2.7) are averaged over a variable volume. Considering the formula for differentiating integrals over a variable volume $\dfrac{d}{dt}\displaystyle\int_{V_i} f_i dV = \int_{V_i}\frac{\partial f_i}{\partial t}dV + \int_{\Sigma} f_i V_{ni} dS$ and the expression of the effective heat capacity in terms of components, as well as the fact that the first two terms are related to the deformability of the skeleton, the following is obtained (nonlinear terms above the second order of smallness are discarded):

$$T\frac{d}{d\tau}\left[K\alpha_T\varepsilon_{kk} + \frac{\partial P}{\partial T}\varepsilon_{kk} + T\frac{\rho_0 C_{ef}}{T_0}\right] = \lambda_q\nabla^2 T - K\varepsilon_{kk}\left[\alpha_{L0}\frac{d\rho_L}{d\tau} + \alpha_{v0}\frac{d\rho_v}{d\tau} + \alpha_{a0}\frac{d\rho_a}{d\tau} + +\alpha_{00}\frac{d\rho_0}{d\tau}\right] -$$

$$-\varepsilon_{kk}\left[\frac{dP}{d\rho_L}\frac{d\rho_L}{d\tau} + \frac{dP}{d\rho_v}\frac{d\rho_v}{d\tau} + \frac{dP}{d\rho_a}\frac{d\rho_a}{d\tau}\right] - \varepsilon_\Phi r\frac{d\rho_v}{d\tau} - J_L\left(C_L T + \frac{P}{\rho_L}\right)\Bigg|_{S_L} -$$

$$-\left(J_a C_{pa} + J_v C_{pv}\right)T\Big|_{S_g} - \sum_i\left(\lambda_i\nabla T\right)\Big|_{S_i} + \left(\lambda\nabla T_g - \lambda\nabla T_0\right)\Big|_{S_{g0}} +$$

$$+\left(\lambda\nabla T_l - \lambda\nabla T_0\right)\Big|_{S_{L0}} + \left(\lambda\nabla T_g - \lambda\nabla T_L\right)\Big|_{S_{Lg}} + \left(C_L - C_{pL}\right)J_L T - \frac{P}{\rho}J_L, \tag{2.8}$$

with the fluxes $J_i = \rho_i \upsilon_i \ \left(i = l, v, \ a\right)$; $J_e = \rho_L\left(\upsilon_{vL} - D_{gL}\right)$; D_{gL} is the velocity of the liquid and gas phases, provided that at the interface (gL): $\upsilon_a = D_{gL}$; $J_e = \rho_v\left(\upsilon_v - D_{gL}\right)$.

In this case, the nonlinear entropy balance equation takes the form:

$$T\frac{d}{d\tau}\left[\left(K\alpha_T + \frac{\partial P}{\partial T}\right)\varepsilon_{kk} + T\frac{\rho_0 C_{ef}}{T_0} + r_0\rho_v\right] = \nabla\left[\lambda_{ef}\nabla T - r_0 J_v -\right.$$

$$-K\varepsilon_{kk}\left[\alpha_{L0}\frac{d\rho_L}{d\tau} + \alpha_{v0}\frac{d\rho_v}{d\tau} + \alpha_{a0}\frac{d\rho_a}{d\tau} + \alpha_{00}\frac{d\rho_0}{d\tau}\right] - \varepsilon_{kk}\left[\frac{dP}{d\rho_L}\frac{d\rho_L}{d\tau} + \frac{dP}{d\rho_v}\frac{d\rho_v}{d\tau} + \frac{dP}{d\rho_a}\frac{d\rho_a}{d\tau}\right] +$$

$$+\left(Q_{g0} - Q_{0g}\right) + \left(Q_{L0} - Q_{0L}\right) + \left(Q_{gL} - Q_{Lg}\right) - T_0\left(K\alpha_T + \frac{\partial P}{\partial T}\right)\frac{\partial \varepsilon_{kk}}{\partial \tau}. \tag{2.9}$$

Here Q_{ij} is the intensity of heat exchange at the interface; $r_0 = r + \left(C_L - C_{pv}\right)T$. The densities included in the heat transfer equation are scaled, not true.

2.2 CONVECTION-DIFFUSION EQUATION FOR MASS TRANSFER

Equation (2.9) includes the time derivatives of ρ_v, ρ_a, ρ_L, which follow from the mass balance equation of the k-th component [31]:

$$\frac{\partial \rho_k}{\partial \tau} + \vec{\nabla} J_k = \sum_{k \neq l} \rho_{kl}, \quad \left(k = L, v, \ a\right), \tag{2.10}$$

where $J_k = \rho_k \vec{\upsilon}_k$, $\vec{\upsilon}_k$ is the velocity of the k-th component of the carrier phase, with ρ_{kl} as the production capacity of the k-th component corresponding to the phase transition of the l-th component to the k-th. These equations can be written as follows:

$$\frac{\partial \rho_k}{\partial \tau} + \vec{\nabla}\cdot\left(D_{kT}\vec{\nabla}T + \left(D_{kP}\vec{\nabla}P\right) + \left(D_{kj}\vec{\nabla}\rho_j\right)\right) = \sum_{k \neq l}\rho_{kl}, \quad \left(k = L, v, \ a\right). \tag{2.11}$$

The heating of the material induces the filtration flow of liquid and gas. The liquid flow is caused by a gradient of liquid concentration in the pores α, temperature T, and pressure of the vapor-air mixture P. The flows of steam and air are caused by the gradients of pressure and mass concentration of steam in the mixture. Assuming that the vapor-air mixture forms a homogeneous phase and the liquid is water, the velocity of the components satisfies Darcy's equation, the fluxes of the components of the carrier phase are presented as follows [31, 32]:

$$\vec{J}_L = -D_L\left(\Pi\rho_L^0\vec{\nabla}\alpha + \rho_0\delta\vec{\nabla}T\right) - \tilde{K}\frac{\rho_L^0}{\mu_L}K_L\vec{\nabla}(P - P_{cap} + \rho_L^0 g),$$

$$\vec{J}_v = -\tilde{K}\frac{\rho_v^0}{\mu_{e\Phi}}K_g\vec{\nabla}P - (1-\alpha)\Pi D_{e\Phi}\rho_g^0\vec{\nabla}\frac{\rho_v^0}{\rho_g^0}, \tag{2.12}$$

$$\vec{J}_a = -\tilde{K}\frac{\rho_a^0}{\mu_{e\Phi}}K_g\vec{\nabla}P + (1-\alpha)\Pi D_{e\Phi}\rho_g^0\vec{\nabla}\frac{\rho_v^0}{\rho_g^0}.$$

Here $D_L, D_{e\Phi}, \tilde{K}, K_g, K_l, \mu_{e\Phi}, P_{cap}, g$ are the effective diffusion coefficients of liquid, gas, permeability, the relative permeability of gas and liquid, effective viscosity of the gas mixture, capillary pressure, acceleration of the earth's gravity; J_i are the flows of liquid, steam, and air; δ is the thermogradient coefficient; $\tilde{K} = \frac{r^2}{1-\Pi}$, with r as the characteristic radius of the pores in the skeleton. The filtration coefficient $\tilde{K}$ depends on the pore size in the sample and the characteristics of the pore space. As a rule, it is assumed that the relative permeability of a substance is proportional to the volume fraction of the substance in the pores [33, 34]: $K_g = 1-\alpha, K_l = \alpha$.

When a liquid phase is present in the pores, zones of entrapped air can occur. In those zones of the material where the air is entrapped, the velocities of liquid and gas are equal and the flows of liquid and vapor are determined by the following relations $J_{a2} = \frac{1-\alpha}{\alpha}\frac{\rho_a}{\rho_L}J_L$, $J_{v2} = \frac{\rho_a}{\rho_L}J_{a2}$, with the first relation J_L being satisfied [35].

Let the transition from the entrapped state to independent phase motion occur when the value α decreases in a certain range of the two-phase zone $\alpha_g \le \alpha \le \alpha_L$, where α_g, α_L depend on the structure of the medium and are considered to be given. To describe the movement of phases in the entire region of moisture content change, the air and vapor flows are presented in the form $J_i = f\,J_{i1} + (1-f)J_{i2}$, $(i = v;a)$, with the continuous function f, along with its derivative, changing from 1 to 0 for $\alpha_g \le \alpha \le \alpha_L$ and being equal to one for $\alpha \le \alpha_g$ and zero for $\alpha \ge \alpha_L$. Let it be expressed in the form:

$$f(\alpha) = 1 - \left[\left(\frac{\alpha-\alpha_g}{\alpha_L-\alpha_g}\right)\sigma_0(\alpha-\alpha_g) - \left(\frac{\alpha-\alpha_L}{\alpha_L-\alpha_g}\right)\sigma_0(\alpha-\alpha_L)\right].$$

In this case, it is possible to study the behavior of the quantities in question at different ratios of flux rates. Summing the first and third equations of system (2.10), the equations of moisture and air transfer are obtained:

$$\frac{\partial}{\partial\tau}\Pi\left[\left(\rho_l^0-\rho_v^0\right)\alpha+\rho_v^0\right]+\nabla J_m = 0, \quad J_m = J_l + J_v,$$

$$\frac{\partial}{\partial\tau}\left[\Pi(1-\alpha)\rho_a^0\right]+\nabla J_a = 0. \tag{2.13}$$

If the porosity is considered constant, then the unknowns in these equations are α and ρ_a. Defining the fluxes of the component velocities as functions of the liquid volume fraction α, pressure P, and mass fraction of vapor in the vapor-air mixture, the following equations will be satisfied:

$$\upsilon_{L1} = \left\{ -D_L \left(\frac{\nabla\alpha}{\alpha} + \frac{\rho_0^0 \delta}{\Pi \alpha \rho_L^0} \nabla T \right) - \frac{\tilde{K}}{\mu_L \Pi} \nabla P \right\},$$

$$\upsilon_{v1} = \left\{ -\frac{D_{ef}}{\rho_v^0 / \rho_g^0} \nabla \left(\rho_v^0 / \rho_g^0 \right) - \frac{\tilde{K}}{\mu_{ef} \Pi} \nabla P \right\},$$

$$\upsilon_{a1} = \left\{ \frac{D_{ef}}{\rho_a^0 / \rho_g^0} \nabla \left(\rho_v^0 / \rho_g^0 \right) - \frac{\tilde{K}}{\mu_{ef} \Pi} \nabla P \right\}.$$

With this representation of the fluxes, the problem of heat and mass transfer can be solved in a one-dimensional formulation. This is possible when the layer thickness is small compared with the length and width. With $\alpha = 0$, the first equation of system (2.13) becomes the equation for determining the density of unsaturated steam ρ_v^0. The value of D_L is determined as in [35]:

$$D_L(T,\alpha) = \begin{cases} \left[D_{L0} + \left(D_{LL} - D_{L0} \right) \sin \dfrac{\pi\alpha}{2\alpha_a} \right] \left(\dfrac{T}{273} \right)^{20}, & \alpha \le \alpha_a, \\ D_{LL} \left(\dfrac{T}{273} \right)^{20}, & \alpha > \alpha_a, \end{cases}$$

where D_{L0}, D_{LL} are the diffusion coefficients of the liquid in the solid skeleton and in the liquid, respectively.

The effective viscosity coefficient of the vapor-air mixture is a function α. Moreover, $\mu_L = \mu_g$, where μ_g is the viscosity of the gas mixture for $0 < \alpha < \alpha_g$, after which μ_{ef} increases from μ_g to μ_L for $\alpha_g < \alpha < \alpha_L$ and remains constant if $\alpha_L < \alpha < 1$. The thermogradient coefficient is also a function α:

$$\delta(\alpha) = \delta_0 \left[1 - 4(\alpha - 0{,}5)^2 \right].$$

In the state of entrapped gas, for $\alpha > \alpha_L$, μ_{ef} and the effective vapor diffusion coefficient D_{ef} are also a function α, with $D_{ef} = D_{\max}$ for $0 < \alpha < \alpha_g$ and decreasing to zero for $\alpha_g < \alpha < \alpha_L$. In the entrapped state, no vapor diffusion occurs in the material $\upsilon_{v2} = \upsilon_{a2} = \upsilon_{12.}$

The vapor-air mixture is considered an ideal gas. Pressure $P_g = \left(\dfrac{\rho^0{}_v}{M_v} + \dfrac{\rho^0{}_a}{M_a} \right) RT$. The density of saturated vapor is a function of temperature and for water is approximated by the Filonenko formula (no hygroscopic state of the material is considered here) $\rho_v^0 = 133\frac{M_v}{RT}\exp\left(18.681-\frac{4105}{T-35}\right)$ when $\alpha > 0$. If the deformation of a solid skeleton is subject to Hooke's law, then linear relationships can be assumed for the fluid, which links overpressure, density, and temperature. The state equation for a fluid can be expressed as $\frac{\rho_L^0}{\rho_{L0}^0} = 1+\beta_{P1}\left(P_L-P_0\right)-\alpha_{TL}\left(T-T_0\right)$. Therefore, $P_{L1} = \frac{1}{\beta_{pL}}\left[\frac{\rho_L^0-\rho_{L0}^0}{\rho_{L0}^0}+\alpha_{TL}\left(T-T_0\right)\right]$ is the overpressure in the fluid. Here, β_{pL}, α_{TL} are the coefficients of volume and temperature expansion, respectively. The overpressure in the carrier phase is $P=\left(1-\alpha\right)P_g+\alpha P_L$. The expression for P_L may include the capillary pressure, which depends on the surface tension.

2.3 COMPATIBILITY EQUATION

If the porosity Π changes during the drying process, it is necessary to have an equation to determine it. Thus, the elasticity relations for a solid skeleton are considered. By Hooke's law for a micro-volume of a solid skeleton:

$$\varepsilon_0'^{kl} = \frac{1}{2\mu_0^\sigma}\left[\sigma_0'^{kl}-\frac{\lambda_0^\sigma}{3\lambda_0^\sigma+2\mu_0^\sigma}\sigma_0'^{mm}\delta^{kl}\right]+1/3\alpha_{T0}\left(T'/T^0-1\right)\delta^{kl}.$$

Let the expression be averaged by convolving it by the indices:

$$\left\langle\varepsilon_0'^{kk}\right\rangle_0 = \frac{1}{2\mu_0^\sigma}\left[\left\langle\sigma_0'^{kk}\right\rangle\frac{1}{3\lambda_0^\sigma+2\mu_0^\sigma}\right]+\alpha_{T0}\left(T'/T^0-1\right)=P_0\frac{1}{3\lambda_0^\sigma+2\mu_0^\sigma}+\alpha_{T0}\frac{T'-T^0}{T^0},$$

$$P_0 = 1/3\left\langle\sigma_0'^{kk}\right\rangle,\quad \left\langle\varepsilon_0'^{kk}\right\rangle_{00} = P_0\frac{1}{3\lambda_0^\sigma+2\mu_0^\sigma},\quad \left\langle\varepsilon_0'^{kk}\right\rangle = \frac{1}{V_0}\int\varepsilon_0'^{kk}dV.$$

The first invariant of the averaged strain tensor $\left\langle\varepsilon_0'^{kk}\right\rangle_0$ determines the change in the true density ρ_0^0: $\left\langle\varepsilon_0'^{kk}\right\rangle_0-\left\langle\varepsilon_0'^{kk}\right\rangle_{00} = -\frac{\rho_0^0-\rho_{00}^0}{\rho_{00}^0}$. For the averaged quantities over the volume of the solution $\varepsilon_0'^{kk}$ is obtained:

$$\varepsilon_0^{kk}-\varepsilon_{00}^{kk} = \frac{\rho_0-\rho_{00}}{\rho_{00}}. \tag{2.14}$$

By Hooke's law for fictitious stresses:

$$P_1 - P_0 = \left[\lambda_f^\sigma + 2/3\mu_f^\sigma\right]\varepsilon_0^{kk} + \nu_f^\sigma P_1 - \nu_f^\sigma\left[\lambda_f^\sigma + 2/3\mu_f^\sigma\right]\alpha_T^0\left(\frac{T}{T_0} - 1\right), \tag{2.15}$$

$$P_1 = \frac{V_H}{V_H + V_0}\left(\alpha_L P_L + \alpha_g P_g\right). \tag{2.16}$$

Differentiating expression (2.15) by τ and considering relation (2.14):

$$-\left[\lambda_f^\sigma + 2/3\mu_f^\sigma\right]\frac{1}{\rho_{00}}\frac{\partial\rho_0}{\partial\tau} - \left(1 - \nu_f^\sigma\right)\frac{\partial P_1}{\partial\tau} + \frac{\partial P_0}{\partial\tau} - \nu_f^\sigma\left[\lambda_f^\sigma + 2/3\mu_f^\sigma\right]\alpha_{T0}/T_0\frac{\partial T}{\partial\tau} = 0. \tag{2.17}$$

By definition, $\rho_0 = \left(1 - \Pi\right)\rho_0^0$. Given that:

$$\frac{\partial P_{1g}}{\partial\tau} = \left(\frac{1}{M_v}\frac{\partial\rho_v^0}{\partial\tau} + \frac{1}{M_a}\frac{\partial\rho_a^0}{\partial\tau}\right)RT + \left(\frac{\rho_v^0}{M_v} + \frac{\rho_a^0}{M_a}\right)R\frac{\partial T}{\partial\tau},$$

$$\frac{\partial P_{1L}}{\partial\tau} = \frac{1}{\beta_{P1}}\left(\frac{1}{\rho_{L0}^0}\frac{\partial\rho_L^0}{\partial\tau} + \frac{\alpha_{TL}}{T_0}\frac{\partial T}{\partial\tau}\right),$$

$$\frac{\partial P_0}{\partial\tau} = \frac{1}{\beta_{P0}}\left(\frac{1}{\rho_{00}^0}\frac{\partial\rho_0^0}{\partial\tau} + \frac{\alpha_{T0}}{T_0}\frac{\partial T}{\partial\tau}\right), \tag{2.18}$$

and by substituting (2.18) into (2.17), the compatibility equation that relates the true densities to the porosity is obtained:

$$-\left[\lambda_f^\sigma + 2/3\mu_f^\sigma\right]\frac{1}{\rho_{00}\beta_{P0}}\left[\frac{\partial}{\partial\tau}\left(1 - \Pi\right)\rho_0^0\right]\frac{\partial P}{\partial\tau} - \left(1 - \nu_f^\sigma\right)\left\{\frac{\alpha}{\beta_{PL}}\left(\frac{1}{\rho_{L0}^0}\frac{\partial\rho_L^0}{\partial\tau} + \frac{\alpha_{TL}}{T_0}\frac{\partial T}{\partial\tau}\right) + \right.$$

$$\left. + \left(1 - \alpha\right)\left[\left(\frac{1}{M_v}\frac{\partial\rho_v^0}{\partial\tau} + \frac{1}{M_a}\frac{\partial\rho_a^0}{\partial\tau}\right)RT + \left(\frac{\rho_v^0}{M_v} + \frac{\rho_a^0}{M_a}\right)R\frac{\partial T}{\partial\tau}\right]\right\} +$$

$$+ \frac{1}{\beta_{P0}}\left(\frac{1}{\rho_{00}^0}\frac{\partial\rho_0^0}{\partial\tau} + \frac{\alpha_{T0}}{T_0}\frac{\partial T}{\partial\tau}\right) = 0. \tag{2.19}$$

2.4 MOMENTUM BALANCE EQUATION

To determine the deformations and average displacements in a solid skeleton, the momentum balance equation for a saturated porous medium is obtained in the form [27]:

$$\rho_1 \frac{d_1 \vec{\upsilon}_1}{d\tau} = -\vec{\nabla}\left(\Pi P\right) - \vec{R}_{10} + \rho_1 \vec{g}_1, \tag{2.20}$$

$$\rho_0 \frac{d_0 \vec{\upsilon}_0}{d\tau} = -\vec{\nabla}\left[\left(1-\Pi\right)P\right] + \vec{\nabla}\cdot\hat{\sigma}_f + \vec{R}_{10} + \rho_0 \vec{g}_0, \tag{2.21}$$

where $\vec{\upsilon}_1 = \frac{\rho_L \vec{\upsilon}_L + \rho_v \vec{\upsilon}_v + \rho_a \vec{\upsilon}_a}{\rho_L + \rho_v + \rho_a}$; $\vec{R}_{10}$ is an interfacial variable. By determining $\vec{R}_{10}$ from equations (2.20) and substituting into (2.21):

$$\rho_0 \frac{d_0 \vec{\upsilon}_0}{d\tau} = -\vec{\nabla}P + \vec{\nabla}\cdot\hat{\sigma}_f - \rho_1 \frac{d_1 \vec{\upsilon}_1}{d\tau} + \rho_1 \vec{g}_1 + \rho_0 \vec{g}_0, \tag{2.22}$$

with

$$\vec{\upsilon}_1 = \frac{\rho_L^0 \left\{-D_L \vec{\nabla}\alpha + \frac{\rho_0^0 \delta}{\Pi \rho_L^0} \vec{\nabla} T\right\} - \frac{\tilde{K}}{\Pi} \vec{\nabla} P \left\{\frac{\alpha \rho_L^0}{\mu_L} + \frac{\left(1-\alpha\right)\left(\rho_v^0 + \rho_a^0\right)}{\mu_{ef}}\right\}}{\alpha \rho_L^0 + \left(1-\alpha\right)\left(\rho_v^0 + \rho_a^0\right)}, \tag{2.23}$$

$$\vec{\upsilon}_0 = \frac{\partial}{\partial\tau}\begin{Bmatrix}\vec{u}_1 \\ \vec{u}_2 \\ \vec{u}_3\end{Bmatrix}, \quad \rho_1 = \Pi\left[\alpha \rho_L + \left(1-\alpha\right)\left(\rho_v^0 + \rho_a^0\right)\right], \tag{2.24}$$

where $\vec{u}_1, \vec{u}_2, \vec{u}_3$ is the displacement vector in the direction of the axes Ox_1, Ox_2, Ox_3.

To solve the system of equations (2.9), (2.13), (2.19), (2.22), it is necessary to set the single-valued conditions for heat and mass transfer, as well as the mechanical and initial conditions. The mechanical conditions at the boundary are given by the surface force vector $\vec{f}$, or the displacement vector $\vec{h}$, or the ratio between the vectors $\vec{f}$ and $\vec{h}$. The condition $\sigma_{ij} n_j = f_i$ reflects the equilibrium of the stresses and forces applied to the boundary distributed over the body volume. For the bearing medium at the boundary, the heat and mass flows of moisture and air are specified. Initial conditions are set at the initial temperature $t = 0$, liquid concentration α, air concentration $\rho_a = P_0 - \frac{\rho_{vs}\left(T_0\right)}{M_v} RT$, and zero initial stresses. Besides, when setting the heat flux in the case of contact drying, $q_e = -\lambda \nabla T + r^* j_v + \rho_p C_p h \frac{\partial T}{\partial \tau}$, where ρ_p, C_p are the specific density and heat capacity of the sieve (the thin plate) on which the grain layer lies. In particular, for a multicomponent inhomogeneous linear viscoelastic body, the momentum balance equations are as follows [27]:

$$\begin{aligned}\rho\frac{\partial^2 u_i}{\partial\tau^2} &= \left[\mu(1-\omega)u_{i,j}\right]_{,j}+\left[\left(\lambda+2/3\mu\omega\right)u_{j,j}\right]_{,i}+\left[\mu(1-\omega)u_{j,i}\right]_{,j}-\\ &-\left[\alpha_T\left(T-T_0\right)+\sum_{\beta}\alpha_\beta\left(\rho_\beta-\rho_{\beta 0}\right)\right]_{,i}\left(\lambda+2/3\mu\right)-\Pi P_{,i}+X_i.\end{aligned} \qquad (2.25)$$

Here, u_i is the displacement, a comma marking the differentiation along the i-th coordinate.

2.5 THE KEY SYSTEM OF EQUATIONS

The obtained relations allow writing a complete system of equations for determining functions $\Pi, \hat{N}, P, \alpha, \rho_v, \rho_a$ and u_i, $\varepsilon_{ij}, \sigma_{ij}\left(i=1,2,3\right)$, namely the heat conduction equation:

$$\begin{aligned}&T/T_0\frac{d}{d\tau}\left[\left(K\alpha_T+\frac{\partial P}{\partial T}\right)\varepsilon_{kk}+\left(1-\Pi\right)\rho_0^0 C_{ef}T+r_0\Pi\left(1-\alpha\right)\rho_v^0\right]=\\ &=\nabla\left[\lambda_{ef}\nabla T-r_0 J_v-\left(C_{pa}J_a+C_{pv}J_v+C_{pL}J_L\right)T-K\varepsilon_{kk}\left[\Pi\alpha_L\frac{d\left(\Pi\alpha\rho_L^0\right)}{d\tau}+\right.\right.\\ &\left.+\Pi\left(1-\alpha\right)\frac{d\left[(1-\alpha)\left(\rho_v^0+\rho_a^0\right)\right]\rho_v}{d\tau}+\left(1-\Pi\right)\frac{d\left[\left(1-\Pi\right)\right]\rho_0^0}{d\tau}\right]-\\ &-\varepsilon_{kk}\left[\frac{dP}{d\rho_L}\frac{d\left(\Pi\alpha\rho_L^0\right)}{d\tau}+\frac{dP}{d\rho_v}\frac{d\left[\Pi\left(1-\alpha\right)\rho_v^0\right]}{d\tau}+\frac{dP}{d\rho_a}\frac{d\left[\Pi\left(1-\alpha\right)\rho_a^0\right]}{d\tau}\right]-T_0\left(K\alpha_T+\frac{\partial P}{\partial T}\right)\frac{\partial\varepsilon_{kk}}{\partial\tau};\end{aligned} \qquad (2.26)$$

– the equation for the pressure:

$$\left(1-\Pi\right)P_1=\Pi\left(1-\alpha\right)P_g+\Pi\alpha P_L, \qquad (2.27)$$

$$P_g=\left(\frac{\rho^0{}_v}{M_v}+\frac{\rho^0{}_a}{M_a}\right)RT, \qquad (2.28)$$

$$P_L=\frac{1}{\beta_{pL}}\left[\frac{\rho_L^0-\rho_{L0}^0}{\rho_{L0}^0}+\alpha_{TL}\left(T-T_0\right)\right], \qquad (2.29)$$

where α_{TL}, β_{pL} are the coefficients of temperature and volume expansion of the liquid;

– the equation for the density of saturated vapor:

$$\rho_v^0 = 133\frac{M_v}{RT}\exp\left(18.681 - \frac{4105}{T-35}\right); \tag{2.30}$$

– the equation of mass balance:

$$\frac{\partial\left[\left(1-\Pi\right)\right]\rho_0^0}{\partial\tau} + \vec{\nabla}\left(\rho_0\upsilon_0\right) = 0, \tag{2.31}$$

$$\alpha_L + \alpha_g = \Pi, \quad \Pi = \frac{V_H}{V_H + V_0}, \tag{2.32}$$

$$\frac{\partial\left[\Pi\left(1-\alpha\right)\right]\rho_a^0}{\partial\tau} + \vec{\nabla}J_a = 0, \tag{2.33}$$

$$\frac{\partial\left[\Pi\left(1-\alpha\right)\right]\rho_v^0}{\partial\tau} + \vec{\nabla}J_v = I_{vL}, \tag{2.34}$$

$$\frac{\partial\left[\Pi\alpha\rho_L^0\right]}{\partial\tau} + \vec{\nabla}J_L = -I_{vL}; \tag{2.35}$$

$$J_L = -D_L\left(\Pi\rho_L^0\nabla\alpha + \rho_0\delta\nabla T\right) - \tilde{K}\frac{\rho_L^0}{\mu_L}K_L\nabla P, \tag{2.36}$$

$$J_v = -\tilde{K}\frac{\rho_v^0}{\mu_{e\Phi}}K_g\nabla P - \left(1-\alpha\right)\Pi D_{e\Phi}\rho_g^0\nabla\frac{\rho_v^0}{\gamma_g^0}, \tag{2.37}$$

$$J_a = -\tilde{K}\frac{\rho_a^0}{\mu_{e\Phi}}K_g\nabla P + \left(1-\alpha\right)\Pi D_{e\Phi}\rho_g^0\nabla\frac{\rho_v^0}{\rho_g^0}. \tag{2.38}$$

Summing the mass balance equations for vapor and liquid, two equations for determining ρ_a^0 and α are obtained:

$$\frac{\partial}{\partial\tau}\Pi\left[\left(\rho_L^0 - \rho_v^0\right)\alpha + \rho_v^0\right] + \nabla J_m = 0, \quad J_m = J_L + J_v, \tag{2.39}$$

$$\frac{\partial}{\partial\tau}\left[\Pi\left(1-\alpha\right)\rho_a^0\right] + \nabla J_a = 0. \tag{2.40}$$

Invariants for the averaged values over the volume of the solution $\varepsilon_0^{kk} - \varepsilon_{00}^{kk} = -\dfrac{\rho_0 - \rho_{00}}{\rho_{00}}$.
The equation of compatibility:

$$-\left[\lambda_f^\sigma + 2/3\mu_f^\sigma\right]\frac{1}{\rho_{00}\beta_{P0}}\left[\frac{\partial}{\partial\tau}(1-\Pi)\rho_0^0\right] - \left(1-\nu_f^\sigma\right)\left\{\frac{\alpha}{\beta_{PL}}\left(\frac{1}{\rho_{L0}^0}\frac{\partial\rho_L^0}{\partial\tau} + \frac{\alpha_{TL}}{T_0}\frac{\partial T}{\partial\tau}\right) + \right.$$

$$\left. + (1-\alpha)\left[\left(\frac{1}{M_v}\frac{\partial\rho_v^0}{\partial\tau} + \frac{1}{M_a}\frac{\partial\rho_a^0}{\partial\tau}\right)RT + \left(\frac{\rho_v^0}{M_v} + \frac{\rho_a^0}{M_a}\right)R\frac{\partial T}{\partial\tau}\right]\right\} + \frac{1}{\beta_{P0}}\left(\frac{1}{\rho_{00}^0}\frac{\partial\rho_0^0}{\partial\tau} + \frac{\alpha_{T0}}{T_0}\frac{\partial T}{\partial\tau}\right) = 0. \quad (2.41)$$

Momentum balance equation:

$$\rho_0\frac{d_0\vec{\upsilon}_0}{d\tau} = -\vec{\nabla}P + \vec{\nabla}\cdot\hat{\sigma}_f - \rho_1\frac{d_1\vec{\upsilon}_1}{d\tau} + \rho_1\vec{g}_1 + \rho_0\vec{g}_0, \quad (2.42)$$

where

$$\vec{\upsilon}_1 = \frac{\rho_L^0\left\{-D_L\vec{\nabla}\alpha + \dfrac{\rho_0^0\delta}{\Pi\rho_L^0}\vec{\nabla}T\right\} - \dfrac{K}{\Pi}\vec{\nabla}P\left\{\dfrac{\alpha\rho_L^0}{\mu_L} + \dfrac{(1-\alpha)\left(\rho_v^0+\rho_a^0\right)}{\mu_{ef}}\right\}}{\alpha\rho_L^0 + (1-\alpha)\left(\rho_v^0 + \rho_a^0\right)}, \quad (2.43)$$

$$\vec{\upsilon}_0 = \frac{\partial}{\partial\tau}\begin{Bmatrix}\vec{u}_1\\ \vec{u}_2\\ \vec{u}_3\end{Bmatrix}, \quad \rho_1 = \Pi\left[\alpha\rho_L + (1-\alpha)\left(\rho_v^0 + \rho_a^0\right)\right], \quad (2.44)$$

$$\sigma_{ij} = \sigma_{ij}^f - P_H\delta^{ij}, \quad (2.45)$$

$$\sigma_{kl}^f = (1-\Pi)\left\{\lambda_f^\sigma\,\varepsilon_0^{mm}\delta^{ikl} + 2\mu_f^\sigma\varepsilon_2^{kl} + \nu_f^\sigma P_1\delta^{kl} - \right.$$

$$\left. -\nu_f^\sigma\left[\lambda_0^\sigma + 2/3\mu_0^\sigma\right]\left[\alpha_{t0}\left(T-T_0\right) + \sum_\beta\alpha_{\beta0}\left(\rho_\beta - \rho_{\beta0}\right)\right]\delta^{ij}\right\}. \quad (2.46)$$

The pressure difference between the carrier and solid phases due to strength:

$$P_1 - P_0 = \left(\lambda_f^\sigma + \frac{2}{3}\mu_f^\sigma\right)\varepsilon_2 + \nu_f^\sigma P_1 - \left[\alpha_f\left(T-T_0\right) + \beta_f\sum_\beta\left(\rho_\beta - \rho_{\beta0}\right)\right], \quad (2.47)$$

where ν_f^σ, α_f, β_f are the effective moduli of elasticity and expansion. Average velocities and strains:

$$\upsilon_0 = \frac{\partial\left\langle u_0\right\rangle_0}{\partial t}, \quad \varepsilon_0'^{kl} = 0,5\left(\frac{\partial u_0'^l}{\partial x^k} + \frac{\partial u_0'^k}{\partial x^l}\right), \quad \varepsilon_0^{kl} = 0,5\left(\frac{\partial\left\langle u_0'^l\right\rangle_0}{\partial x^k} + \frac{\partial\left\langle u_0'^k\right\rangle_0}{\partial x^l}\right), \tag{2.48}$$

$$\varepsilon_0^{kl} = \left\langle\varepsilon_0^{kl}\right\rangle_0 + \varepsilon_f^{kl},$$

$$\varepsilon_{kl}^f = -0,5\left[\left\langle u_0'^l\right\rangle_0 \frac{\nabla^k(1-\Pi)}{(1-\Pi)} + \left\langle u_0'^k\right\rangle_0 \frac{\nabla^l(1-\Pi)}{(1-\Pi)}\right] - \sum_j \frac{S_{0j}}{2(1-\Pi)}\left\langle u_0'^l u_0'^k + u_0'^k u_0'^l\right\rangle_{0j}, \tag{2.49}$$

where S_{0j} is the inner surface of the porous body.

2.6 NUMERICAL EXPERIMENT

Let's consider a thin plane plate, a surface of which from one side is subjected to the external heat flow $q_e(t)$ (**Fig. 2.1**).

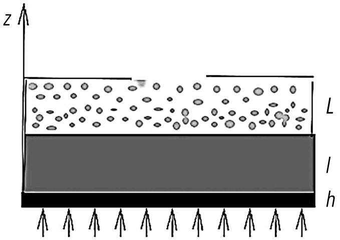


Fig. 2.1 Schematic representation of the model

Conductive contact drying takes place in a steam-air (gas) environment by transferring heat to the material when it is in contact with heated surfaces. The plate has an area s, thickness h_w, its material is characterized by density ρ_w, specific heat capacity C_w. A layer of capillary-porous moisture-saturated material of the thickness l is placed on this plate. The capillary-porous material has the porosity Π, density ρ_0^0, specific heat capacity C_0, and thermal conductivity coefficient in the dry state λ_0.

From the open side of the capillary-porous material, the moisture evaporates into the cavity of the volume V and the depth $L = V/S$. There is an outlet in the cavity through which the steam-air mixture flows into the environment under pressure P_e. The cavity is thermally insulated. It is possible to neglect the heat capacity of its walls. Such an installation can serve as an example of a drying chamber for conductive drying.

The system of heat and mass transfer equations is described as follows:

$$T/T_0\frac{d}{d\tau}\left[\rho_0 C_{ef} T + r_0\Pi\left(1-\alpha\right)\rho_v^0\right] = \nabla\left[\lambda_{ef}\nabla T - r_0 J_v - \sum_{i=l,v,a} C_{pi} T J_i\right], \tag{2.50}$$

$$P_g = \left(\frac{\rho^0{}_v}{M_v} + \frac{\rho^0{}_a}{M_a}\right)RT,$$

$$\Pi\frac{\partial\left[\left(\rho_L^0 - \rho_v^0\right)\alpha + \rho_v^0\right]}{\partial\tau} + \nabla J_m = 0, \tag{2.51}$$

$$J_L + J_v = J_m,$$

$$\Pi\frac{\partial\left[\left(1-\alpha\right)\rho_a^0\right]}{\partial\tau} + \vec{\nabla} J_a = 0, \tag{2.52}$$

$$\rho_v^0 = 133\frac{M_v}{RT}\exp\left(18{,}681 - \frac{4105}{T-35}\right). \tag{2.53}$$

Here Π, α, r_0, λ_{ef}, J_i are the porosity, relative moisture saturation, specific heat of vaporization, effective thermal conductivity, and moisture, steam, and air flows, respectively. If the evaporation is not strong, then it can be roughly assumed that the steam pressure in the cavity is equal to the saturation pressure. In this system of equations, the temperature T, moisture saturation α, and air density ρ_a^0 are unknown. At the initial moment, there can be moisture, air, steam in the pores. It is possible to assume that the steam-air mixture is a mixture of ideal gases and in the wet state, when the capillary-porous material is saturated with moisture $\alpha > 0$, the density of steam-air mixture is a function of temperature only. The equation does not include the phase transition criterion, the dependence of which on the parameters is complex. The equations remain valid in the dry zone, where there is no moisture, and $\alpha = 0$, $I_v = 0$ in this domain, equation (2.52) serves to determine the moisture density.

The boundary conditions are formulated as follows: at the initial moment of time, the pressure of the steam-air mixture in the capillary-porous material and in the cavity is equal to the external atmospheric pressure P_e: $P_g = P_e(0) = P_0$.

The initial temperature:

$$T(x,0) = T_0. \tag{2.54}$$

The moisture saturation $\alpha(x, 0) = \alpha_0 \leq 1$.

The air density $\rho_a^0\left(x,0\right) = \frac{P_0 - P_{vs}}{RT_0}M_a$.

The boundary conditions on the side of the heated plate are as follows:

$$q_e = -\lambda \nabla T + \rho_w C_w h_w \frac{\partial T}{\partial \tau} + r^* J_v,\ r^*(T) = r_0 - \left(C_L - C_{pv}\right) T. \tag{2.55}$$

The moisture and air flows at the interface from the side of the plate are zero:

$$J_m = 0,\ J_a = 0.$$

The boundary conditions on the surface of the capillary-porous material from the side of the cavity with the opening for $x = l$ are as follows:

$$-\lambda \nabla T = r * J_L + V / S\left(\rho_a^0 C_{va} + \rho_v^0 C_{vv}\right) \frac{\partial T}{\partial t} + \frac{V}{S}\left[R_a \frac{\partial \rho_a^0}{\partial t} + R_v \frac{\partial \rho_v^0}{\partial t}\right] T, \tag{2.56}$$

where the first term $\lambda \nabla T$ characterizes the heat flow that penetrates inside the body; the second term is equal to the product of the specific heat of vaporization multiplied by the density of the moisture flow that evaporates; the third term is the power spent on heating the surface; the fourth term is the flow of heat transmitted by the movement of the steam-air mixture.

The total flow of vaporized moisture should be equal to the flow rate of the moisture flowing out through the hole, to estimate which let's use the formula for adiabatic output from the cavity [36]. To determine the flow of moisture, the equation of conservation of moisture mass in the cavity is used:

$$SJ_m = Q_e \frac{\rho_v^0}{\rho_g^0} + V \frac{\partial \rho_v^0}{\partial t}, \tag{2.57}$$

the air flow:

$$SJ_a = Q_e \frac{\rho_a^0}{\rho_g^0} + V \frac{\partial \rho_a^0}{\partial t},\ \rho_v^0 = \rho_{vs}\left(T_c\right). \tag{2.58}$$

The vapor density is equal to the saturated vapor density. The movement of gas in the cavity into which evaporation occurs is neglected. The gas temperature in the cavity is assumed to be the same throughout the volume.

The flow of the steam-air mixture through the drainage hole is determined by the formulae of output from the cavity:

$$Q_e = s\left[\gamma \left(\frac{2}{\gamma + 1}\right)^{\frac{\gamma+1}{\gamma-1}}\right]^{1/2} Q\left(\varepsilon_p\right) P_c \left(R_g T_c\right)^{-1/2},\ \varepsilon_p = P_e / P_c, \tag{2.59}$$

$$Q\left(\varepsilon_p\right)=\begin{cases}\left[\left[\dfrac{\gamma+1}{2}\right]^{\frac{1}{\gamma-1}}\varepsilon_p^{\,1/\gamma}\left[\dfrac{\gamma+1}{\gamma-1}\left(1-\varepsilon_p^{\frac{\gamma-1}{\gamma}}\right)\right]\right]^{1/2}, \ \varepsilon_p>\left[\dfrac{2}{\gamma+1}\right]^{\frac{\gamma}{\gamma-1}}, \\ 1, \ \varepsilon_p\le\left[\dfrac{2}{\gamma+1}\right]^{\frac{\gamma}{\gamma-1}}.\end{cases}$$

Here T_c, P_c are temperature and pressure in the cavity, Q_e is the gas flow through the drainage hole [37], γ is the adiabatic index, R_g is a gas constant. The boundary conditions are obtained under the assumption that the gradients of temperature, pressure, and concentration across the cavity are negligible, and the vapor pressure in the cavity is close to the saturation pressure for the cavity temperature.

Let's write the system of nonlinear differential equations (2.1)–(2.3) in a matrix form:

$$\frac{\partial}{\partial T}\left[E\left(u\right)\right]+\frac{\partial}{\partial x}\vec{J}=0, \tag{2.60}$$

where $u=\left(T,\alpha,\rho_a^0\right)$; $\vec{E}$ is a vector, the components of which are the total content of enthalpy, moisture, and air in a unit volume of the material; $\vec{J}$ is a vector composed of heat, moisture, and air flows, it is linearly related to the gradients T, α, P; $c_a=\rho_a^0/\rho_g^0$; $J\left(u\right)=-A\left(u\right)\dfrac{\partial F\left(u\right)}{\partial x}$; $F=F\left(T,\alpha,P,c_a\right)$; $A(u)$ is the 3×4 matrix, and the 3×5 matrix (if capillary pressure is taken into account) [38–40]:

$$A=\left[a_{ij}\right],\ i=\overline{1,4},\ j=\overline{1,3},$$

where $a_{11}=\lambda$, $a_{12}=0$; $a_{13}=\dfrac{\tilde{K}}{\mu_{eff}}\left(1-\alpha\right)\left(r_0\rho_v^0+TC_{pa}\rho_a^0+TC_{pv}\rho_v^0\right)$;

$$a_{14}=\Pi D_{ef}\left(1-\alpha\right)\left(\rho_v^0+\rho_a^0\right)\left(r_0-C_{pa}T-C_{pv}T\right);\quad a_{21}=a_L\rho_0\delta;\quad a_{22}=D_L\Pi\rho_L^0;$$

$$a_{23}=\alpha\frac{\tilde{K}\rho_L^0}{\mu_L}+\frac{\tilde{K}\left(1-\alpha\right)}{\mu_{ef}}\rho_v^0;\quad a_{24}=\Pi D_{ef}\left(1-\alpha\right)\left(\rho_v^0+\rho_a^0\right);\quad a_{31}=0;$$

$$a_{32}=0;\quad a_{33}=\frac{\tilde{K}\left(1-\alpha\right)}{\mu_{ef}}\rho_a^0;\quad a_{34}=-\Pi D_{ef}\left(1-\alpha\right)\left(\rho_v^0+\rho_a^0\right).$$

Here D_{ef}, C_{pa}, C_{pv} are the coefficients of effective diffusion, specific heat capacities of air and steam at a constant pressure, respectively; λ is the coefficient of effective thermal conductivity:

$$E\left(T,\alpha,\rho_a^0\right)=\left|\begin{matrix} T\Big\{\rho_0 C_0+\Pi\Big[\rho^0{}_L C_L\alpha+\left(1-\alpha\right)\Big(\rho_v^0 C_{vv}+\rho_a^0 C_{va}\Big)\Big\}+r_0\Pi\left(1-\alpha\right)\rho_v^0\Big] \\ \Pi\Big[\left(\rho_L^0-\rho_v^0\right)\alpha+\rho_v^0\Big] \\ \Pi\left(1-\alpha\right)\rho_v^0 \end{matrix}\right|;$$

$$A=\left|\begin{matrix} \lambda & 0 & \dfrac{\tilde{K}K_g}{\mu_{ef}}\Big(r_0\rho_v^0+T(C_{pa}\rho_a^0+C_{pv}\rho_v^0)\Big) & \Pi D_{ef}\left(1-\alpha\right)\rho_g^0\Big(r_0-(C_{pa}+C_{pv})T\Big) \\ D_L\rho_0\delta & D_L\rho^0{}_L\Pi & \left[\dfrac{\tilde{K}K_L\rho_L^0}{\mu_L}+\dfrac{\tilde{K}K_g\rho_v^0}{\mu_{ef}}\right]+ & \Pi D_{ef}\left(1-\alpha\right)\left(\rho_v^0+\rho_a^0\right) \\ 0 & 0 & \dfrac{\tilde{K}K_g\rho_a^0}{\mu_{ef}} & -\Pi D_{ef}\left(1-\alpha\right)\left(\rho_v^0+\rho_a^0\right) \end{matrix}\right|;$$

where $K_g=1-\alpha$ is a relative gas permeability; r_0 is the heat of vaporization for $T=0$ K;

$$F=\left|\begin{matrix} T \\ \alpha \\ \left(\dfrac{\rho^0{}_v}{M_v}+\dfrac{\rho^0{}_a}{M_a}\right)RT \\ \dfrac{\rho_v^0}{\rho_v^0+\rho_a^0} \end{matrix}\right|.$$

The system of equations has to satisfy the boundary conditions:

$$J\big|_{x=0}\left(u\right)=Q_0,\ J\big|_{x=1}\left(u\right)=Q_1,$$

and the initial conditions: $(0\le\left(x\right)\le l,\ t=0),\ T=T_0,\ \alpha=\alpha_0<1,$

$$\rho_a^0=\left[P_0-P_{vs}\left(T\right)\right]M_a/RT_0, \tag{2.61}$$

$$Q_0=\left|\begin{matrix} q_e-\rho_p C_p h\partial T/\partial t \\ 0 \\ 0 \end{matrix}\right|$$ is the gas flow from the side of the plate,

$$Q_1 = \left| \begin{array}{l} \frac{V}{S}\left\{ \left(\frac{\partial \rho_a^0}{\partial t} C_{va} + \frac{\partial \rho_v^0}{\partial T}\frac{\partial T}{\partial t} C_{vv} \right) T + \left(\rho_a^0 C_{va} + \rho_v^0 C_{vv} \right) \frac{\partial T}{\partial t} + r_0 \frac{\partial \rho_v^0}{\partial T}\frac{\partial T}{\partial t} \right\} + \\ + \frac{Q_e}{S}\frac{\left(\rho_a^0 C_{va} + \rho_v^0 C_{vv} \right) T + r_0 \rho_v^0}{\rho_a^0 + \rho_v^0} \\ \frac{1}{S}\frac{Q_e \rho_v^0}{\rho_a^0 + \rho_v^0} + \frac{V}{S}\frac{\partial \rho_v^0}{\partial T}\frac{\partial T}{\partial t} \\ \frac{1}{S}\frac{Q_e \rho_v^0}{\rho_a^0 + \rho_v^0} + \frac{V}{S}\frac{\partial \rho_a^0}{\partial T} \end{array} \right| .$$

The nonlinear problem is solved by two methods for comparing the results.

Construction of a difference scheme.

Let's integrate the matrix Eq. (2.60) with respect to x over the interval $x_n - \frac{\Delta x}{2}, \quad x_n + \frac{\Delta x}{2}$ for $t = t_k = k\Delta t$. Let's obtain:

$$-\int\limits_{x_n + \frac{\Delta x}{2}}^{x_n - \frac{\Delta x}{2}} \frac{\partial E^k}{\partial t} dx + J^k_{n+\frac{1}{2}} - J^k_{n-\frac{1}{2}} = 0. \tag{2.62}$$

Equation (2.62) after the difference approximation is reduced to the difference scheme. A three-point approximation of the spatial variables is used. The system of nonlinear algebraic equations is solved by Newton's method.

The linearization method.

In order to solve the boundary value problem, in addition, an iterative process is built, at each step of which a linear boundary value problem is solved for the next approximation, which uses the information of the previous one. A small-time step is used to ensure convergence of iterations.

If an approximation $u_i = \begin{pmatrix} T \\ \alpha \\ \rho_a^0 \end{pmatrix}$ of the problem solution is known, then the exact solution u^* can be presented as follows $u^* = u_i + \Delta u_i^*$.

Put $u^* = u_{i+1}$.

Based on the Lagrange formula:

$$\dot{E}\left[u_{i+1}(t,x) \right] = \dot{E}\left[u_i(t,x) \right] + \left\{ \left[\dot{E}\left[u_i(t,x) \right] \right]'_T, \left[\dot{E}\left[u_i(t,x) \right] \right]'_\alpha, \left[\dot{E}\left[u_i(t,x) \right] \right]'_{\rho_a} \right\} \times$$

$$\times\begin{bmatrix} T_{i+1}-T_i \\ \alpha_{i+1}-\alpha_i \\ \rho_{ai+1}-\rho_{ai} \end{bmatrix} + \left\{ \left[\dot{E}\left[u_i(t,x)\right]\right]'_{\dot{T}}, \left[\dot{E}\left[u_i(t,x)\right]\right]'_{\dot{\alpha}}, \left[\dot{E}\left[u_i(t,x)\right]\right]'_{\dot{\rho}_a} \right\} \begin{bmatrix} \dot{T}_{i+1}-\dot{T}_i \\ \dot{\alpha}_{i+1}-\dot{\alpha}_i \\ \dot{\rho}_{ai+1}-\dot{\rho}_{ai} \end{bmatrix}, \qquad (2.63)$$

$$J(t,x,u_{i+1}) = -A(t,x,u_{i+1})\frac{\partial F}{\partial x}(t,x,u_{i+1},\dot{u}_{i+1}) = J\left[u_i(t,x)\right] + \left\{ J\left[u_i(t,x)\right]'_T, J\left[u_i(t,x)\right]'_\alpha,\right.$$

$$\left. J\left[u_i(t,x)\right]'_{\rho_a} \right\} \begin{bmatrix} T_{i+1}-T_i \\ \alpha_{i+1}-\alpha_i \\ \rho_{ai+1}-\rho_{ai} \end{bmatrix} + \left\{ J\left[u_i(t,x)\right]'_{T'}, J\left[u_i(t,x)\right]'_{\alpha'}, J\left[u_i(t,x)\right]'_{\rho'_a} \right\} \begin{bmatrix} T'_{i+1}-T'_i \\ \alpha'_{i+1}-\alpha'_i \\ \rho'_{ai+1}-\rho'_{ai} \end{bmatrix}. \qquad (2.64)$$

Using quadrature formulae of the interpolation type according to the 3/8 rule [38], let's obtain the difference scheme:

$$1/8\left(\frac{\Delta E^k_{n-1}}{\Delta t} + 6\frac{\Delta E^k_n}{\Delta t} + \frac{\Delta E^k_{n+1}}{\Delta t}\right) + \frac{1}{\Delta x}\left(J^k_{n+1/2} - J^k_{n-1/2}\right) = 0, \qquad (2.65)$$

where $\frac{\Delta E^k_n}{\Delta t} = \frac{1}{\Delta t}\left[E^k_n - E^{k-1}_n\right]$, $E^k_n = E\left(u^k_n\right)$,

$$\{J\}'_{T/\alpha/\rho_a} = -\left\{ \left[A\right]'_{T/\alpha/\rho_a}\left[\frac{\partial F}{\partial x}\right] + \left[A\right]\left[\frac{\partial F}{\partial x}\right]'_{T/\alpha/\rho_a} \right\},$$

$$\{J\}'_{T'/\alpha'/\rho'_a} = -\left\{ \left[A\right]\left[\frac{\partial F}{\partial x}\right]'_{T'/\alpha'/\rho'} \right\}, \qquad (2.66)$$

by $\left[\ \right]'_{T,\alpha,\rho_a}$ the differentiations with respect to T, α, ρ_a are denoted.

Denote

$$\left[\frac{\partial E}{\partial t}(u,t,x)\right]_{u_i,t_k,x_n} = \left[\dot{E}^k_{in}\right],\ \left[J(u,t,x)\right]_{u_i,t_k,x_n} = \left[J^k_{in}\right], \qquad (2.67)$$

$$J_i\left(t^k, x_{n+1/2}\right) = -\frac{1}{2}\left[\left(A^k_n + A^k_{n+1}\right)\frac{F^k_{n+1} - F^k_n}{\Delta x}\right]_i.$$

Taking into account Eqs. (2.63), (2.64) and the boundary conditions:

$$1/8\left(3\frac{\Delta E_0^k}{\Delta t}+\frac{\Delta E_1^k}{\Delta t}\right)+\frac{1}{\Delta x}\left(J_{1/2}^k-Q_0\right)=0,$$

$$1/8\left(3\frac{\Delta E_N^k}{\Delta t}+\frac{\Delta E_{N-1}^k}{\Delta t}\right)+\frac{1}{\Delta x}\left(Q_1-J_{N-1/2}^k\right)=0, \qquad (2.68)$$

arrive at the iterative scheme of linear equations. If the *i*-th iteration of the solution u_{in}^k is known, then using Lagrange's formula $E_{i+1,n}^k=E_{in}^k+E_{1in}^k\left(u_{i,n}^k-u_{in}^k\right)$, let's obtain:

$$I_{i+1,n+1/2}^k=I_{i,n+1/2}^k+I_{1in+1/2}^k\left(u_{i+1,n+1}^k-u_{in+1}^k\right)+I_{2i,n+1/2}^k\left(u_{i+1,n}^k-u_{in}^k\right).$$

Here E_{in}^k, $I_{i,n+1/2}^k$ are values of the vectors $\vec{E}_i$, $\vec{I}_i$ at the points (n,k), $(n+1/2,k)$.

To verify the result, let's apply a slightly modified method of linearization, which is less time-consuming for the difference scheme. It is possible to proceed from equations (2.60), (2.61), (2.65)–(2.68), where $Q_0=\left(q_{h0,}\,q_{m0},\,q_{a0}\right)$ are the flows of enthalpy, moisture, and air through the surface $x = 0$. Then $q_{m0}=0$, $q_{a0}=0$, $q_{h0}=q_1-\rho_p c_p h\Delta T_0^k/\Delta t$, N is the number of nodes on x; $Q_1=\left(q_{h1,}\,q_{m1},\,q_{a1}\right)$ are flows through the surface $x=l$:

$$q_{h1}=\frac{V}{S}\frac{\Delta}{\Delta t}\left[\left(\rho_{aN}c_{va}+\rho_{vN}c_{vv}\right)T_N+r_0\rho_{vN}^k\right]+\frac{Q_e}{S\left(1+X_N\right)}\left[\left(X_N c_{pa}+c_{pv}\right)T_N+r_0\right];$$

$$q_{m1}=\frac{V}{S}\frac{\Delta}{\Delta t}\rho_{vN}+\frac{Q_e}{S\left(1+X_N\right)};$$

$$q_{a1}=\frac{V}{S}\frac{\Delta}{\Delta t}\rho_{aN}+\frac{Q_e X_N}{S\left(1+X_N\right)}. \qquad (2.69)$$

Based on Lagrange's formula, let's present

$$E_{i+1n}^k=E_{in}^k+E_{1in}^k\left(u_{i+1,n}^k-u_{i,n}^k\right), \qquad (2.70)$$

$$J_{i+1,n+1/2}^k=J_{in+1/2}^k+J_{1in+1/2}^k\left(u_{i+1,n+1}^k-u_{i,n+1}^k\right)+J_{2in+1/2}^k\left(u_{i+1,n}^k-u_{i,n}^k\right),$$

where E_{in}^k, $J_{in+1/2}^k$ the values of the vectors $\vec{E}_i$, $\vec{J}_i$ at the points (n,k), $(n+1/2,k)$, respectively. E_{1in}^k, $J_{1in+1/2}^k$, $J_{2in+1/2}^k$ are matrices formed as follows:

$$E^k_{1in} = \left[\frac{\partial E^k_n}{\partial T^k_n}, \frac{\partial E^k_n}{\partial \alpha^k_n}, \frac{\partial E^k_n}{\partial \rho^{\ k}_{an}} \right]_i, \quad J^k_{1in+1/2} = \left[\frac{\partial J^k_{n+1/2}}{\partial T^k_{n+1}}, \frac{\partial J^k_{n+1/2}}{\partial \alpha^k_{n+1}}, \frac{\partial J^k_{n+1/2}}{\partial \rho^{\ k}_{an+1}} \right]_i,$$

$$J^k_{2in+1/2} = \left[\frac{\partial J^k_{n+1/2}}{\partial T^k_n}, \frac{\partial J^k_{n+1/2}}{\partial \alpha^k_n}, \frac{\partial J^k_{n+1/2}}{\partial \rho^{\ k}_{an}} \right]_i. \tag{2.71}$$

By analogy:

$$J^k_{i+1,n-1/2} = J^k_{in-1/2} + J^k_{1in-1/2}\left(u^k_{i+1,n-1} - u^k_{i,n-1}\right) + J^k_{2in-1/2}\left(u^k_{i+1,n} - u^k_{i,n}\right),$$

where

$$J^k_{in-1/2} = \left[J^k_{in-1/2} \right],$$

$$J^k_{in+1/2} = \left[J^k_{in+1/2} \right],$$

$$J^k_{1in-1/2} = \left[\frac{\partial J^k_{n-1/2}}{\partial T^k_{n-1}}, \frac{\partial J^k_{n-1/2}}{\partial \alpha^k_{n-1}}, \frac{\partial J^k_{n-1/2}}{\partial \rho^{\ k}_{an-1}} \right]_i,$$

$$J^k_{2in-1/2} = \left[\frac{\partial J^k_{n-1/2}}{\partial T^k_n}, \frac{\partial J^k_{n-1/2}}{\partial \alpha^k_n}, \frac{\partial J^k_{n-1/2}}{\partial \rho^{\ k}_{an}} \right]_i,$$

where

$$\frac{\partial J^k_{n+1/2}}{\partial T^k_{n+1} / \alpha^k_{n+1} / \rho^{\ k}_{an+1}} = -\frac{1}{2}\left\{ \frac{\partial A\left(u^k_{n+1}\right)}{\partial T^k_{n+1} / \alpha^k_{n+1} / \rho^{\ k}_{an+1}} \left[\frac{F^k_{n+1} - F^k_n}{\Delta x} \right] - \left[A^k_n + A^k_{n+1} \right] \frac{1}{\Delta x} \frac{\partial F^k_n}{\partial T^k_{n+1} / \alpha^k_{n+1} / \rho^{\ k}_{an+1}} \right\},$$

$$\frac{\partial J^k_{n+1/2}}{\partial T^k_n / \alpha^k_n / \rho^{\ k}_{an}} = -\frac{1}{2}\left\{ \frac{\partial A\left(u^k_n\right)}{\partial T^k_n / \alpha^k_n / \rho^{\ k}_{an}} \left[\frac{F^k_{n+1} - F^k_n}{\Delta x} \right] - \left[A^k_n + A^k_{n+1} \right] \frac{1}{\Delta x} \frac{\partial F^k_n}{\partial T^k_n / \alpha^k_n / \rho^{\ k}_{an}} \right\},$$

$$\frac{\partial J^k_{n-1/2}}{\partial T^k_{n-1} / \alpha^k_{n-1} / \rho^{\ k}_{an-1}} = -\frac{1}{2}\left\{ \frac{\partial A\left(u^k_{n-1}\right)}{\partial T^k_{n-1} / \alpha^k_{n-1} / \rho^{\ k}_{an-1}} \left[\frac{F^k_{n+1} - F^k_n}{\Delta x} \right] - \left[A^k_n + A^k_{n-1} \right] \frac{1}{\Delta x} \frac{\partial F^k_n}{\partial T^k_{n-1} / \alpha^k_{n-1} / \rho^{\ k}_{an-1}} \right\},$$

$$\frac{\partial J^k_{n-1/2}}{\partial T^k_n / \alpha^k_n / \rho^{\ k}_{an}} = -\frac{1}{2}\left\{ \frac{\partial A\left(u^k_n\right)}{\partial T^k_n / \alpha^k_n / \rho^{\ k}_{an}} \left[\frac{F^k_n - F^k_{n-1}}{\Delta x} \right] - \left[A^k_n + A^k_{n-1} \right] \frac{1}{\Delta x} \frac{\partial F^k_n}{\partial T^k_n / \alpha^k_n / \rho^{\ k}_{an}} \right\}.$$

The variables Q_0, Q_1 are presented in the form:

$$\left[Q_0\right]_{i+1,0}^{k}=\left[Q_0\right]_{i,0}^{k}+\left[Q_0^1\right]_{i,0}^{k}\left(u_{i+1,0}^{k}-u_{i,0}^{k}\right)+\left[Q_0^2\right]_{i,0}^{k}\left(u_{i+1,0}^{k-1}-u_{i,0}^{k-1}\right),$$

$$\left[Q_0\right]_{i+1,N}^{k}=\left[Q_0\right]_{i,N}^{k}+\left[Q_1^1\right]_{i,N}^{k}\left(u_{i+1,N}^{k}-u_{i,N}^{k}\right)+\left[Q_1^2\right]_{i,N}^{k}\left(u_{i+1,N}^{k-1}-u_{i,N}^{k-1}\right), \tag{2.72}$$

where

$$\left[Q_0^1\right]_{i,0}^{k}=\left[\frac{\partial\left[Q_0\right]_{i,0}^{k}}{\partial T_0^k},0,0\right]_i,$$

$$\left[Q_0^2\right]_{i,0}^{k}=\left[\frac{\partial\left[Q_0\right]_{i,0}^{k}}{\partial T_0^{k-1}},0,0\right]_i,$$

$$\left[Q_1^1\right]_{i,N}^{k}=\left[\frac{\partial\left[Q_1\right]_{i,N}^{k}}{\partial T_N^k},\frac{\partial\left[Q_1\right]_{i,N}^{k}}{\partial \alpha_N^k},\frac{\partial\left[Q_1\right]_{i,N}^{k}}{\partial \rho_{aN}^k}\right]_i,$$

$$\left[Q_1^2\right]_{i,N}^{k}=\left[\frac{\partial\left[Q_1\right]_{i,N}^{k}}{\partial T_N^{k-1}},\frac{\partial\left[Q_1\right]_{i,N}^{k}}{\partial \alpha_N^{k-1}},\frac{\partial\left[Q_1\right]_{i,N}^{k}}{\partial \rho_{aN}^{k-1}}\right]_i.$$

Taking into account these ratios, equations (2.65)–(2.68) are written as follows:

$$\left[\frac{3}{8\Delta t}E_{1i0}^{k}+\frac{1}{\Delta x}\left[J_{2i\frac{1}{2}}^{k}-\left(Q_0^1\right)_{i0}^{k}\right]\right](u_{i+10}^{k}-u_{i0}^{k})-\left[\frac{3}{8\Delta t}E_{1i0}^{k-1}+\frac{1}{\Delta x}\left[\left(Q_0^2\right)_{i0}^{k}\right]\right](u_{i+10}^{k-1}-u_{i0}^{k-1})+$$
$$+\left[\frac{1}{8\Delta t}E_{1i1}^{k}+\frac{1}{\Delta x}J_{1i\frac{1}{2}}^{k}\right](u_{i+11}^{k}-u_{i1}^{k})-\frac{1}{8\Delta t}E_{1i1}^{k-1}(u_{i+11}^{k-1}-u_{i1}^{k-1})=-\frac{1}{8\Delta t}\left[3\left(E_{10}^{k}-E_{10}^{k-1}\right)+\right.$$
$$\left.+\left(E_{11}^{k}-E_{11}^{k-1}\right)\right]-\frac{1}{\Delta x}\left[J_{i\frac{1}{2}}^{k}-\left(Q_0\right)_{i0}^{k}\right];$$

$$\left[\frac{1}{8\Delta t}E_{1in-1}^{k}-\frac{1}{\Delta x}J_{1in-\frac{1}{2}}^{k}\right](u_{i+1n-1}^{k}-u_{in-1}^{k})-\frac{1}{8\Delta t}E_{1in-1}^{k-1}(u_{i+1n-1}^{k}-u_{in-1}^{k})+$$
$$+\left[\frac{6}{8\Delta t}E_{1in}^{k}+\frac{1}{\Delta x}\left[J_{2in+\frac{1}{2}}^{k}-J_{2in-\frac{1}{2}}^{k}\right]\right](u_{i+1n}^{k}-u_{in}^{k})-\frac{6}{8\Delta t}E_{1in}^{k-1}(u_{i+1n}^{k-1}-u_{in-1}^{k-1})+$$

$$+\left[\frac{1}{8\Delta t}E^{k}_{1in+1}+\frac{1}{\Delta x}J^{k}_{1in+\frac{1}{2}}\right](u^{k}_{i+1n+1}-u^{k}_{in+1})-\frac{1}{8\Delta t}E^{k-1}_{1in+1}(u^{k-1}_{i+1n+1}-u^{k-1}_{in+1})=$$

$$=-\frac{1}{8\Delta t}\left[6\left(E^{k}_{in}-E^{k-1}_{in}\right)+\left(E^{k}_{in-1}-E^{k-1}_{in-1}\right)+\left(E^{k}_{in+1}-E^{k-1}_{in+1}\right)\right]-\frac{1}{\Delta x}\left[J^{k}_{in+\frac{1}{2}}-J^{k}_{in-\frac{1}{2}}\right]; \qquad (2.73)$$

where

$$\left[Q_0\right]^{k}_{i0}=\left|\begin{matrix} q_e-\rho_p c_p h\dfrac{T^{k}_{0}-T^{k-1}_{0}}{\Delta t} \\ 0 \\ 0 \end{matrix}\right|_{i},$$

$$Q_1=\left[b_i\right],\ i=\overline{1,3},$$

$$b_1=\frac{V}{S}\left\{\left(\frac{\partial\rho^{0}_{a}}{\partial t}C_{va}+\frac{\partial\rho^{0}_{v}}{\partial T}\frac{\partial T}{\partial t}C_{vv}\right)T+\left(\rho^{0}_{a}C_{va}+\rho^{0}_{v}C_{vv}\right)\frac{\partial T}{\partial t}+r_0\frac{\partial\rho^{0}_{v}}{\partial T}\frac{\partial T}{\partial t}\right\}+$$

$$+\frac{Q_e}{S}\frac{\left(\rho^{0}_{a}C_{va}+\rho^{0}_{v}C_{vv}\right)T+r_0\rho^{0}_{v}}{\left(\rho^{0}_{a}+\rho^{0}_{v}\right)};$$

$$b_2=\frac{1}{S}\frac{Q_e\rho^{0}_{v}}{\rho^{0}_{v}+\rho^{0}_{a}}+\frac{V}{S}\frac{\partial\rho^{0}_{v}}{\partial T}\frac{\partial T}{\partial t};$$

$$b_3=\frac{1}{S}\frac{Q_e\rho^{0}_{a}}{\rho^{0}_{v}+\rho^{0}_{a}}+\frac{V}{S}\frac{\partial\rho^{0}_{a}}{\partial t};$$

$$\left[Q_1\right]^{K}_{iN}=\left|\begin{matrix} \dfrac{V}{S}\left\{C_{va}T^{k}_{N}\dfrac{\rho^{0k}_{aN}-\rho^{0k-1}_{aN}}{\Delta t}+\left[\left(C_{vv}T+r_0\right)\dfrac{\partial\rho^{0}_{v}}{\partial T}+\left(\rho^{0}_{a}C_{va}+\rho^{0}_{v}C_{vv}\right)\right]^{k}_{N}\dfrac{T^{k}_{N}-T^{k-1}_{N}}{\Delta t}\right\}+ \\ +\dfrac{Q_e}{S}\dfrac{\left(\rho^{0}_{a}C_{va}+\rho^{0}_{v}C_{vv}\right)T+r_0\rho^{0}_{v}}{\left(\rho^{0}_{a}+\rho^{0}_{v}\right)}\Bigg|^{k}_{N} \\ \dfrac{Q_e}{S}\dfrac{\rho^{0}_{v}}{\left(\rho^{0}_{a}+\rho^{0}_{v}\right)}\Bigg|^{k}_{N}+\dfrac{V}{S}\dfrac{\partial\rho^{0}_{v}}{\partial T}\Bigg|^{k}_{N}\dfrac{T^{k}_{N}-T^{k-1}_{N}}{\Delta t} \\ \dfrac{Q_e}{S}\dfrac{\rho^{0}_{v}}{\left(\rho^{0}_{a}+\rho^{0}_{v}\right)}\Bigg|^{k}_{N}+\dfrac{V}{S}\dfrac{\rho^{0k}_{aN}-\rho^{0k-1}_{aN}}{\Delta t} \end{matrix}\right|_{i},$$

$$\frac{\partial Q_{1N}^{k}}{\partial T_N^k}=\left|\begin{array}{l}\frac{V}{S}\left\{C_{va}\frac{\rho_{aN}^{0k}-\rho_{aN}^{0k-1}}{\Delta t}+\left[\left(C_{vv}T+r_0\right)\frac{\partial\rho_v^0}{\partial T}+\left(\rho_a^0C_{va}+\rho_v^0C_{vv}\right)\right]_N^k\frac{1}{\Delta t}\right\}+\\ +\frac{T_N^k-T_N^{k-1}}{\Delta t}\left[\frac{\partial^2\rho_v^0}{\partial T^2}\left(C_{vv}T+r_0\right)+2C_{vv}\frac{\partial\rho_v^0}{\partial T}\right]_N^k+\frac{Q_e}{S}\left\{\left[\frac{\left(\rho_a^0C_{va}+\rho_v^0C_{vv}\right)T+r_0\rho_v^0}{\left(\rho_a^0+\rho_v^0\right)^2}+\right.\right.\\ \left.\left.+\frac{\left(C_{pv}T+r_0\right)}{\left(\rho_a^0+\rho_v^0\right)}\right]\frac{\partial\rho_v^0}{\partial T}\Big|_N^k+\frac{\left(\rho_a^0C_{pa}+\rho_v^0C_{pv}\right)}{\left(\rho_a^0+\rho_v^0\right)}\Big|_N^k\right\}\\ \frac{Q_e}{S}\frac{\rho_v^0}{\left(\rho_a^0+\rho_v^0\right)^2}\frac{\partial\rho_v^0}{\partial T}\Big|_N^k+\frac{V}{S}\frac{\partial\rho_v^0}{\partial T}\Big|_N^k\frac{1}{\Delta t}+\frac{T_N^k-T_N^{k-1}}{\Delta t}\left(\frac{\partial^2\rho_v^0}{\partial T^2}\right)_N^k\\ -\frac{Q_e}{S}\frac{\rho_a^0}{\left(\rho_a^0+\rho_v^0\right)^2}\frac{\partial\rho_v^0}{\partial T}\Big|_N^k\end{array}\right|_i,$$

$$\frac{\partial Q_{1N}^{k}}{\partial\alpha_N^k}=0,\quad\frac{\partial Q_{1N}^{k}}{\partial\alpha_N^{k-1}}=0,$$

$$\frac{\partial Q_{1N}^{k}}{\partial T_N^{k-1}}=\left|\begin{array}{l}-\frac{V}{S}\left[\left(C_{vv}T+r_0\right)\frac{\partial\rho_v^0}{\partial T}+\left(\rho_a^0C_{va}+\rho_v^0C_{vv}\right)\right]_N^k\frac{1}{\Delta t}\\ -\frac{V}{S}\frac{\partial\rho_v^0}{\partial T}\Big|_N^k\frac{1}{\Delta t}\\ 0\end{array}\right|;$$

$$\frac{\partial Q_{1N}^{k}}{\partial\rho_{aN}^k}=\left|\begin{array}{l}\frac{V}{S}\left\{C_{va}T_N^k\frac{1}{\Delta t}+C_{va}\frac{T_N^k-T_N^{k-1}}{\Delta t}\right\}+\frac{Q_e}{S}\left[\frac{\left(C_{pa}T\right)}{\left(\rho_a^0+\rho_v^0\right)}-\frac{\left(\rho_a^0C_{pa}+\rho_v^0C_{pv}\right)T+r_0\rho_v^0}{\left(\rho_a^0+\rho_v^0\right)^2}\right]_N^k\\ -\frac{Q_e}{S}\frac{\rho_v^0}{\left(\rho_a^0+\rho_v^0\right)^2}\Big|_N^k\\ -\frac{Q_e}{S}\frac{\rho_v^0}{\left(\rho_a^0+\rho_v^0\right)^2}\Big|_N^k+\frac{V}{S}\frac{1}{\Delta t}\end{array}\right|;$$

$$\frac{\partial Q_{1N}^{\,k}}{\partial \rho_{aN}^{\,k-1}} = \left| \begin{matrix} -\dfrac{V}{S} C_{va} T_N^k \dfrac{1}{\Delta t} \\ 0 \\ \dfrac{1}{\Delta t} \end{matrix} \right|;$$

$$\frac{\partial Q_{00}^{\,k}}{\partial T_0^k} = \left| \begin{matrix} -\rho_p C_p h \dfrac{1}{\Delta t}\Big|_0^k \\ 0 \\ 0 \end{matrix} \right|_i ; \quad \frac{\partial Q_{00}^{\,k}}{\partial T_0^{k-1}} = \left| \begin{matrix} \rho_p C_p h \dfrac{1}{\Delta t}\Big|_0^{k-1} \\ 0 \\ 0 \end{matrix} \right|;$$

$$\frac{\partial Q_{00}^{\,k}}{\partial \alpha_0^k} = 0; \quad \frac{\partial Q_{00}^{\,k}}{\partial \alpha_0^{k-1}} = 0; \quad \frac{\partial Q_{00}^{\,k}}{\partial \rho_{a0}^{\,k}} = 0; \quad \frac{\partial Q_{00}^{\,k}}{\partial \rho_{a0}^{\,k-1}} = 0;$$

$$\frac{\partial E_N^k}{\partial \alpha_N^k} = \left| \begin{matrix} \left\{ T\Pi\left[C_L \rho_L - \left(\rho_a^0 C_{va} + \rho_v^0 C_{vv} \right) \right] - r_0 \Pi \rho_v^0 \right\}_N^k \\ -\Pi\left(\rho_L^0 - \rho_v^0 \right)\Big|_N^k \\ -\Pi \rho_a^0 \Big|_N^k \end{matrix} \right|,$$

$$\frac{\partial E_N^k}{\partial T_N^k} = \left| \begin{matrix} \left\{ \rho_0 C_0 + \Pi\left[C_L \rho_L \alpha + (1-\alpha)\left(\rho_a^0 C_{va} + \rho_v^0 C_{vv} \right) \right]_N^k \right\} + \Pi(1-\alpha)\left(TC_{vv} + r_0 \right) \dfrac{\partial \rho_v^0}{\partial T}\Big|_N^k \\ \Pi(1-\alpha) \dfrac{\partial \rho_v^0}{\partial T}\Big|_N^k \\ 0 \end{matrix} \right|,$$

$$\frac{\partial E_N^k}{\partial \rho_{aN}^{\,k}} = \left| \begin{matrix} T\Pi(1-\alpha) C_{va} \Big|_N^k \left\{ \rho_0 C_0 + \Pi\left[C_L \rho_L \alpha + (1-\alpha)\left(\rho_a^0 C_{va} + \rho_v^0 C_{vv} \right) \right]_N^k \right\} + \\ + \Pi(1-\alpha)\left(TC_{vv} + r_0 \right) \dfrac{\partial \rho_v^0}{\partial T}\Big|_N^k \\ 0 \\ \Pi(1-\alpha)\Big|_N^k \end{matrix} \right|,$$

$$F=\begin{vmatrix} T \\ \alpha \\ \left(\dfrac{\rho_v^0}{\rho_v^0+\rho_a^0}\right) \\ \dfrac{\rho_v^0}{\rho_v^0+\rho_a^0} \end{vmatrix}.$$

$$\frac{\partial A_N^k}{\partial T_N^k}=\begin{vmatrix} 0 & 0 & \dfrac{\tilde{K}}{\mu_{eff}}(1-\alpha)\Big[\left(r_0\rho_v^0+TC_{pa}\rho_a^0+TC_{pv}\rho_v^0\right)+ & \Pi D_{ef}\left(1-\alpha\right)\left(\rho_v^0+\rho_a^0\right)\left(C_{pv}-C_{pa}\right)+ \\ & & +C_{pa}\rho_a^0+C_{pv}\rho_v^0+\left(TC_{pv}+r_0\right)\dfrac{\partial\rho_v^0}{\partial T}\Big]_N^k & +\dfrac{\partial\rho_v^0}{\partial T}\left(r_0-C_{pa}T+C_{pv}T\right)\Big|_N^k \\ \dfrac{\partial D_L}{\partial T}\rho_0\delta & \dfrac{\partial D_L}{\partial T}\rho_L & \dfrac{\tilde{K}}{\mu_{eff}}\left(1-\alpha\right)\dfrac{\partial\rho_v^0}{\partial T}\Big|_N^k & \Pi D_{ef}\left(1-\alpha\right)\dfrac{\partial\rho_v^0}{\partial T}\Big|_N^k \\ 0 & 0 & 0 & \Pi D_{ef}\dfrac{\tilde{K}}{\mu_{eff}}\left(1-\alpha\right)\rho_v^0\Big|_N^k - \end{vmatrix};$$

$$\frac{\partial A_N^k}{\partial \alpha_N^k}=\begin{vmatrix} \dfrac{\partial\lambda}{\partial\alpha} & 0 & -\dfrac{\tilde{K}}{\mu_{eff}}\Big[\left(r_0\rho_v^0+TC_{pa}\rho_a^0+TC_{pv}\rho_v^0\right)+ & -\Pi D_{ef}\left(\rho_v^0+\rho_a^0\right)\left(r_0-C_{pa}T+C_{pv}T\right) \\ & & +C_{pa}\rho_a^0+C_{pv}\rho_v^0++\left(TC_{pv}+r_0\right)\dfrac{\partial\rho_v^0}{\partial T}\Big]_N^k & +\dfrac{\partial\rho_v^0}{\partial T}\left(r_0-C_{pa}T+C_{pv}T\right)\Big|_N^k \\ \left(\dfrac{\partial D_L}{\partial\alpha}\rho_0\delta+\right. & \dfrac{\partial D_L}{\partial\alpha}\Pi\rho_L & \dfrac{\tilde{K}\rho_L}{\mu_L}-\dfrac{\tilde{K}\rho_v^0}{\mu_{ef}}\Big|_N^k & -\Pi D_{ef}\left(\rho_v^0+\rho_a^0\right)\Big|_N^k \\ \left.+a_L\rho_0\dfrac{\partial\delta}{\partial\alpha}\dfrac{\partial a_L}{\partial\alpha}\right)\Big|_N^k & & & \\ 0 & 0 & -\dfrac{\tilde{K}\rho_a^0}{\mu_{ef}}\Big|_N^k & \Pi D_{ef}\left(\rho_v^0+\rho_a^0\right)\Big|_N^k \end{vmatrix};$$

$$\frac{\partial A_N^k}{\partial \rho_N^k}=\begin{vmatrix} 0 & 0 & -\dfrac{\tilde{K}}{\mu_{eff}}(1-\alpha)Tc_{pa} & -\Pi D_{ef}(1-\alpha)\left(r_0-C_{pa}T+C_{pv}T\right) \\ 0 & 0 & 0 & \Pi D_{ef}(1-\alpha) \\ 0 & 0 & \dfrac{\tilde{K}}{\mu_{ef}}(1-\alpha)\Big|_N^k & -\Pi D_{ef}(1-\alpha)\Big|_N^k \end{vmatrix};$$

$$\left[\frac{3}{8\Delta t}E_{1iN}^{k}-\frac{1}{\Delta x}\left[J_{2iN-\frac{1}{2}}^{k}-\left(Q_{1}^{1}\right)_{iN}^{k}\right]\right]\left(u_{i+1N}^{k}-u_{iN}^{k}\right)-\left[\frac{3}{8\Delta t}E_{1iN}^{k-1}+\frac{1}{\Delta x}\left[\left(Q_{1}^{1}\right)_{iN}^{k}\right]\right]\left(u_{i+1N}^{k-1}-u_{iN}^{k-1}\right)+$$

$$+\left[\frac{1}{8\Delta t}E_{1iN-1}^{k}-\frac{1}{\Delta x}J_{1iN-\frac{1}{2}}^{k}\right]\left(u_{i+1N-1}^{k}-u_{iN-1}^{k}\right)\;-\frac{1}{8\Delta t}E_{1iN-1}^{k-1}\left(u_{i+1N-1}^{k-1}-u_{iN-1}^{k-1}\right)=$$

$$=-\frac{1}{8\Delta t}\left[3\left(E_{1N}^{k}-E_{1N}^{k-1}\right)+\left(E_{1N-1}^{k}-E_{1N-1}^{k-1}\right)\right]-\frac{1}{\Delta x}\left[J_{iN-\frac{1}{2}}^{k}+\left(Q_{1}\right)_{iN}^{k}\right].$$

The transfer coefficients are taken from the Lykov's work [32]. This model describes mass transfer processes under moderate heat loads.

The capillary-porous material for which:

$$\lambda=\begin{cases}\lambda_0+\left(\lambda_L-\lambda_0\right)\sin\dfrac{\pi\alpha}{2\alpha_\lambda},\ \alpha\le\alpha_\lambda,\\ \lambda_L,\ \alpha>\alpha_\lambda.\end{cases}$$

Here $\lambda_L = 0.06$ W/(m K), $\alpha_\lambda = 0.1$;

$$\frac{\partial\lambda}{\partial\alpha}=\begin{cases}\dfrac{\pi}{2\alpha_\lambda}\left(\lambda_L-\lambda_0\right)\cos\dfrac{\pi\alpha}{2\alpha_\lambda},\ \alpha<\alpha_\lambda,\\ 0,\ \alpha>\alpha_\lambda;\end{cases}$$

$$D_L\left(T,\alpha\right)=\begin{cases}\left[D_{L0}+\left(D_{LL}-D_{L0}\right)\sin\dfrac{\pi\alpha}{2\alpha_a}\right]\left(T/273\right)^{20},\ \alpha<\alpha_D,\\ D_{LL}\left(T/273\right)^{20},\ \alpha>\alpha_D;\end{cases}$$

$$\frac{\partial D_L\left(T,\alpha\right)}{\partial T}=\begin{cases}\left[D_{L0}+\left(D_{LL}-D_{L0}\right)\sin\dfrac{\pi\alpha}{2\alpha_a}\right]\left(20T^{19}/273^{20}\right),\ \alpha<\alpha_D,\\ D_{LL}\left(20T^{19}/273^{20}\right),\ \alpha<\alpha_D;\end{cases}$$

$$\frac{\partial D_L\left(T,\alpha\right)}{\partial\alpha}=\begin{cases}\dfrac{\pi}{2\alpha_a}\left(D_{LL}-D_{L0}\right)\cos\dfrac{\pi\alpha}{2\alpha_a}\Bigg]\left(T/273\right)^{20},\ \alpha<\alpha_D,\\ 0,\ \alpha<\alpha_D;\end{cases}$$

$$\delta\left(\alpha\right)=\delta_0\left[1-4\left(\alpha-0{,}5\right)^2\right],$$

$$\frac{\partial\delta(\alpha)}{\partial\alpha}=\delta_0(4-8\alpha);$$

$$\frac{\partial\rho_v^0}{\partial T}=133\frac{M_v}{RT}\exp\left(18.681-\frac{4105}{T-35}\right)\left[\frac{4105}{(T-35)^2}-\frac{1}{T}\right],$$

$$\frac{\partial^2\rho_v^0}{\partial T^2}=133\frac{M_v}{RT}\exp\left(18.681-\frac{4105}{T-35}\right)\left[\left(\frac{4105}{(T-35)^2}-\frac{1}{T}\right)^2-\left(\frac{2\cdot 4105}{(T-35)^3}-\frac{1}{T^2}\right)\right];$$

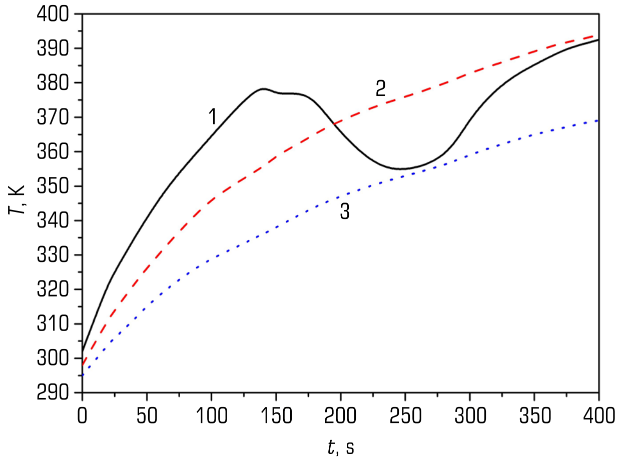
$$P_c=\left(\frac{\rho_v^0}{M_v}+\frac{\rho_a^0}{M_a}\right)TR,\ \frac{\partial P_c}{\partial\rho_v^0}=\frac{RT}{M_v},\ \frac{\partial P_c}{\partial\rho_a^0}=\frac{RT}{M_a},\ \frac{\partial P_c}{\partial T}=\left(\frac{\rho_v^0}{M_v}+\frac{\rho_a^0}{M_a}\right)R.$$

Calculations are made for an aluminum plate and a capillary-porous material of different porosity (cork tree). The flow of the continuous phase is assumed to be slow. Inertial terms are neglected. Transfer coefficients are considered to be known functions of saturation and temperature.

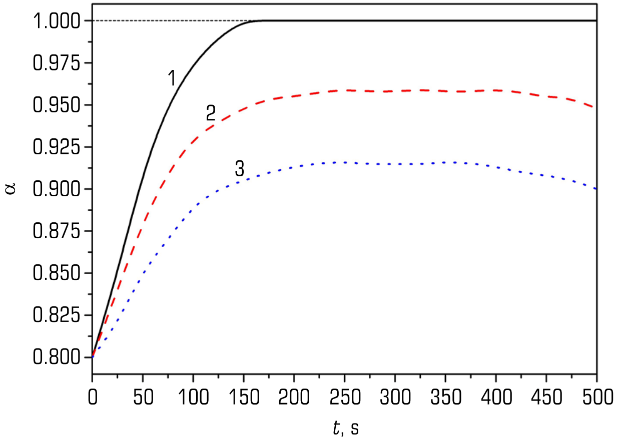
$T_0 = 290$ K, $h = 2\cdot10^{-3}$ m, $D_{LL} = 1.5D_{L0}$, $\alpha_0 = 0.2$, $R = 1.01325\cdot10^5$ Pa, $T_0 = 8.31$ J/Kmol, $M_v = 1.8\cdot10^{-3}$ kg/mol, $M_a = 2.9\cdot10^{-3}$ kg/mol, $c_{pa} = 1.006\cdot10^3$ J/(kg·K), $c_{pv} = 1.103\cdot10^3$ J/(kg·K), $C_{va} = 718$ J/(kg·K), $C_{vv} = 862$ J/(kg·K), $\alpha_L=9.5\cdot10^{-1}$, $r_0^* = 2.3\cdot10^6$ J/kg, $P_e = 10$ Pa, $l = 5\cdot10^{-2}$ m, $L=3\cdot10^{-2}$ m, $\Pi=9\cdot10^{-1}$, $C_L = 4.190\cdot10^3$ J/(kg·K), $C_0 = 10^3$ J/(kg·K), $\tilde{K}=10^{-14}\,m^2$, $\mu_L = 5\cdot10^{-4}$ kg/(m·s), $\mu_{ef} = \mu_g = 10^{-5}$ kg/(m·s), $D_{ef} = 5\cdot10^{-5}$ m²/s, $\lambda_0 = 6\cdot10^{-2}$ W/(m·K), $\lambda_L = 6\cdot10^{-1}$ W/(m·K), $\alpha_g = 8.5\cdot10^{-1}$, $\alpha_\lambda = \alpha_a = 10^{-1}$, $\rho_0 = 6\cdot10^{-2}$ kg/m³, $\rho_L = 10^3$ kg/m³, $\delta_0 = 10^{-3}$ 1/K, $D_{L0} = 10^{-3}$ m²/s, $T_c = 327$ K, $s/S = 10^{-4}$, $V/S = 3\cdot10^{-2}$ m.

As an example, porous materials with the porosity $\Pi = 0.4$, 0.6, and 0.8 heated by heat flows $q = 3\cdot10^3$, $5\cdot10^3$, 10^4 are considered and the influence of various parameters on drying processes is investigated. The results of the calculations are shown in **Fig. 2.2–2.7**.

The solutions of the problem are obtained by finite-difference and iterative methods, and the comparison of the results of these solutions is used to study their accuracy. Calculations have shown that, depending on the magnitude of the heat flux, porosity, and initial saturation of the capillary-porous material, evaporation proceeds differently. The temperature (dependent on porosity) under the action of the flow $q = 3\cdot10^3$, $5\cdot10^3$, 10^4 W/m² with the porosity $\Pi = 0.4$, 0.6 during 500 s is monotonically increasing function of time, but for $q = 10^4$ W/m² and $\Pi = 0.8$, this dependence is no longer monotonous either inside or on the surfaces of the material. With a heat flux $q = 10^4$ W/m² and the porosity $\Pi = 0.8$, already at the 150th second of drying, a moisture of a certain mass is released from the material (condensation caused by oncoming warm and cold flows), while the temperature first decreases slightly and then increases with time slower than in a material with the same characteristics but with less porosity. In this case, the lower the porosity, the greater the gradient of temperature rise. This property is used in problems of thermal protection of materials. With the same porosity and heat flow at the beginning of the evaporation process, the temperature increases faster with a lower initial moisture content of the material.



⬡ **Fig. 2.2** Temperature variations in time for $\Pi = 0.9$ and $\alpha_0 = 0.8$. The curves 1, 2, 3 correspond to the $q = 10^4$, $5 \cdot 10^3$, $3 \cdot 10^3$, respectively



⬡ **Fig. 2.3** Change in volumetric saturation in time on the outer surface for $\alpha_0 = 0.8$. The curves 1, 2, 3 correspond to the $q = 3 \cdot 10^3$, $5 \cdot 10^3$, 10^4, respectively

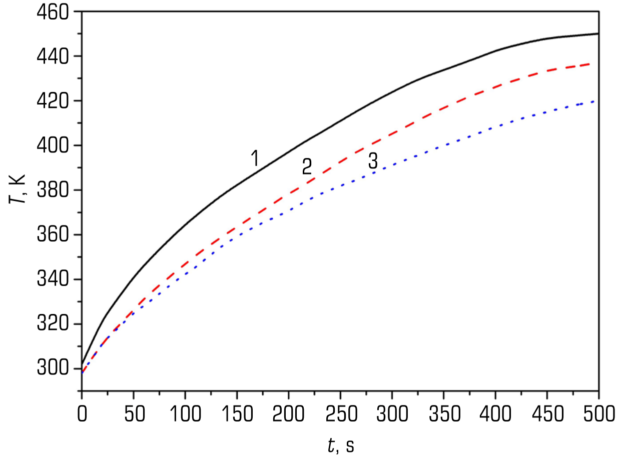


Fig. 2.4 Temperature variations in time under the action of the flow $q = 5{\cdot}10^3$. The curves 1, 2, 3 correspond to the porosity $\Pi = 0.4, 0.6, 0.8$, respectively

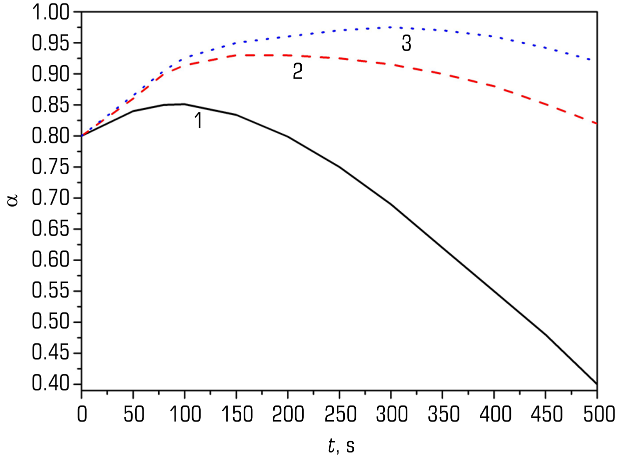


Fig. 2.5 Change in volumetric saturation in time on the outer surface for $q = 5{\cdot}10^3$, $\alpha_0 = 0.8$. The curves 1, 2, 3 correspond to the porosity $\Pi = 0.4, 0.6, 0.8$, respectively

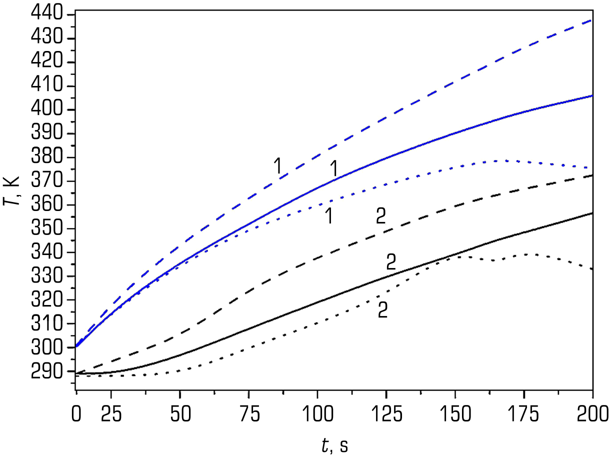


Fig. 2.6 Temperature variations in time on the heating surface (curves 1); external surface (curves 2) for $q = 10^4$, $\alpha_0 = 0.8$ for different values of porosity (dashed curves for $\Pi = 0.4$; solid curves for $\Pi = 0.6$, dotted curves for $\Pi = 0.8$)

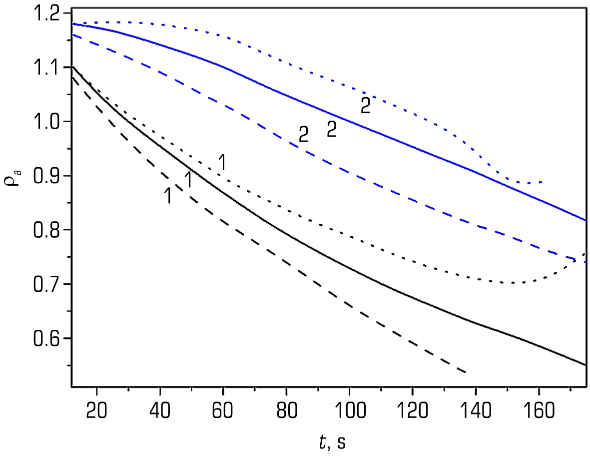


Fig. 2.7 Change in air density in time on the heating surface (curves 1); external surface (curves 2) for $q = 10^4$, $\alpha_0 = 0.8$ for different values of porosity (dashed curves for $\Pi = 0.4$; solid curves for $\Pi = 0.6$, dotted curves for $\Pi = 0.8$)

CONCLUSIONS

The basic statements are formulated and fundamental thermodynamic relations for moisturized capillary-porous deformable systems are obtained when describing them using continuum representations. Possible methods of choosing the parameters of the local thermodynamic state of a solid deformable multi-component system are presented, being consistent with their choice of the liquid (gaseous) phase. A complete system of equations is constructed to describe the drying process of dense packing of capillary-porous materials, based on the approaches of the theory of the mixtures of porous and dense packing of disperse materials of multicomponent three-phase media. There have been analyzed the influence of the external heat flow, the initial volumetric moisture saturation on changes in temperature, volumetric moisture saturation, and air density in body pores in time by the example of conductive drying. The magnitude of the heat flows of the external environment and the initial relative moisture saturation during contact drying of the material affects the behavior in time of both the temperature and the saturation of the porous solid. These characteristics are especially important in the first drying stage when the influence of the initial conditions is important. Therefore, the phenomena that occur at the heating stage with a large initial moisture content were considered.

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