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1 OPTIMIZING CROP ROTATIONS FOR SUSTAINABLE LAND MANAGEMENT AND RESILIENCE OF THE AGRICULTURAL SECTOR OF ECONOMY

ABSTRACT

Ensuring the resilience of any national economy depends significantly on the strength of its constituent sectors. Given the agricultural sector's dominant share in Ukraine's GDP and foreign exchange earnings from exports, the resilience of the Ukrainian economy depends to a large extent on the resilience and sustainability of its agricultural sector. In turn, the resilience and sustainability of Ukraine's agricultural sector are contingent upon its ability to withstand environmental, military and other threats. Soil degradation poses a threat to food and environmental security, as well as to ecological resilience and the achievement of certain Sustainable Development Goals. Although soil degradation is a serious challenge, proactive land-use management can not only mitigate its vulnerability to climate change, but also prevent and avert soil degradation, reduce the risk of erosion and ensure improved resilience for agricultural producers. One aspect of this proactive management approach should be the optimization of crop rotation planning. In this study, a bibliometric landscape of the global knowledge base on agricultural resilience was first established as a theoretical foundation for ensuring the resilience of Ukraine's agricultural sector. An economic-mathematical model was then formulated for crop rotation planning using a combinatorial approach. Finally, solutions to the crop rotation optimization problems were proposed, taking into account the insufficient constraints on environmental conditions. The proposed economic-mathematical model and the results of crop rotation optimization will be useful (1) for policymakers when developing and implementing agricultural and environmental policies regarding the optimal crop mix in crop rotations, and (2) for managers of agricultural enterprises when planning cropland areas and making management decisions.

KEYWORDS

Resilience of agroecosystems, resilience of agriculture, sustainable soil management, crop rotation planning, diversification, modelling project solutions.

The relevance, importance and significance of ensuring the sustainability and resilience of the agricultural sector in the current context of development stem, on the one hand, from the sector's importance to any country and, on the other hand, from the uncertainty and intensification of global environmental, economic, social and political challenges, as well as from global instability. According to a search of the Scopus database as of 31 March 2026, 92,984 published documents containing the term "resilience" in their titles have been indexed, with around 58% of these having been published within the last five years (2021–2025), indicating a significant increase in academic interest in this issue.

Various approaches to defining resilience can be found in the literature. Drawing on the research [1, 2], this paper defines the resilience of the agricultural sector of the economy as its ability to withstand external disturbances and global shocks (for example, the negative impact of climate change, market fluctuations, war), and to recover through internal adaptive mechanisms and proactive measures.

Agricultural and food systems must ensure food security, livelihoods for the millions of people working in this sector, and environmental sustainability. The agricultural sector is therefore far too important to rely on a "reactive" approach to tackling shocks after they have occurred. Instead, it is important to adopt a proactive approach and build resilience, that is to be prepared to absorb, recover, and successfully adapt and transform in response to adverse events. Appropriate policies strengthen the resilience of agricultural producers and other actors in food systems, and invest in prevention and early warning, rather than relying on 'ex post' support after a shock has occurred [3].

The optimization of crop rotations should be one of the strategic measures of a proactive policy aimed at ensuring sustainable land management and the resilience of the agricultural sector. This study therefore aims to provide an economic and mathematical justification for the optimization of crop rotations in agricultural enterprises in Ukraine.

In the first stage, a bibliometric landscape of the global knowledge base on agricultural resilience was established (based on an analysis of publications from the Scopus database using the VOSvier programme) as a theoretical foundation for ensuring the resilience of Ukraine's agricultural sector. In the second stage, an economic-mathematical model was formulated regarding the design of crop rotation using a combinatorial approach. The third stage proposes a solution to the problems of crop rotation optimization, taking into account the lack of constraints on environmental conditions. The economic-mathematical model of optimizing crop rotations over time and constructing model (reference) variants for agricultural enterprises in Ukraine was solved using the LPSolve IDE v.5.5.2.0 programme and Python v. 3.9.0 code for non-commercial use. Optimization of crop allocation over time in accordance with the set task showed that the best combinatorial arrangements for three- to seven-field crop rotations are two crops: spring barley and grain maize. The models found to be optimal from an economic perspective were undiversified crop rotations with low levels of biodiversity, a practice commonly adopted by large Ukrainian agricultural enterprises. However, this does not ensure an adequate level of erosion control, which necessitates the inclusion of environmental criteria in crop rotation optimization models.

1.1. AGRICULTURAL RESILIENCE: A BIBLIOMETRIC LANDSCAPE

The results of the study showed that between 1994 and the first quarter of 2026, Scopus indexed 15,325 published documents containing the phrases "resilience" and "agriculture" in the title, abstract and/or keywords (TITLE-ABS-KEY), of which 458 documents (2.99%) featured this phrase in the title, i.e. directly relate to agricultural resilience. According to the Scopus database, these terms first appeared in the title of a publication in 1994; subsequently, up until 2010, the annual number of documents did not exceed two; between 2011 and 2015, no more than 10 documents were indexed each year. Since 2016, there has been a clear upward trend in publication activity in this field (**Fig. 1.1**), which peaked in 2025, when 139 documents (30.3% of the total) containing the terms "resilience" and "agriculture" in their titles were indexed. Therefore, based on data from the Scopus database, it can be noted that intensive research into agricultural resilience has taken place over the last 10 years, as 83.6% of the documents containing the terms "resilience" and "agriculture" in their titles were published between 2016 and 2025.

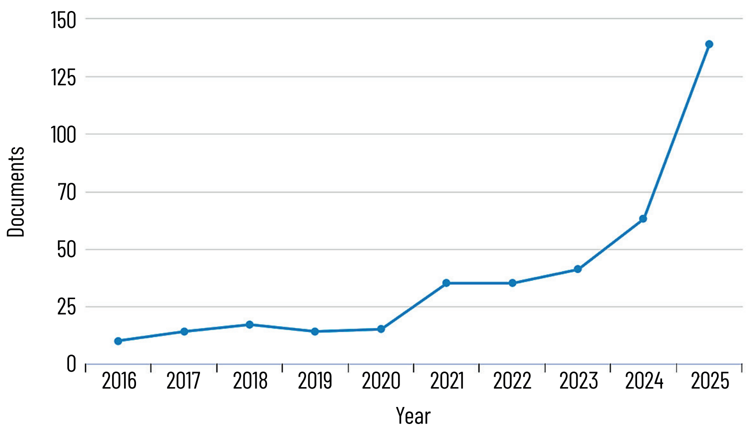

Fig. 1.1 Global trends in the number of Scopus-indexed publications containing the terms "resilience" and "agriculture" in their titles

If the current rate of change in publication activity regarding agricultural resilience continues, it is highly likely that global scientific output will double in the near future.

An analysis of the distribution of documents by field of study revealed that the largest number of published works fell within Agricultural and Biological Sciences – 213 documents, or 22% of the total, Environmental Science – 205 documents, or 21.1% of the total; and Social Sciences – 122 documents, or 12.6% of the total. Other leading fields include Engineering – 92 documents (9.2%) and Energy – 49 documents (5.1%). As regards the economic sciences, 45 documents fall within the field of Economics, Econometrics and Finance, accounting for 4.6%, while 31 documents are categorized under Business, Management and Accounting, accounting for 3.2%. Thus, the economic sciences account for 7.8% of all published documents on agricultural resilience. Overall, the wide range of disciplines covered by the published documents indicates that the issue of agricultural resilience is multidisciplinary.

An analysis of the distribution of indexed documents by publication source – that is, scientific journals, shows that the leading journals include: Sustainability (20 documents), Frontiers in Sustainable Food Systems (9), Agricultural Systems (6), and Frontiers in Plant Science (6).

The results of the analysis of the distribution of documents by type show that 210 works were published as journal articles, accounting for 45.9%, 94 works as book chapters (20.5%), 89 works were reviews (19.4%), 34 were conference papers (7.4%) and 9 works were books (2%). The metadata of these 436 works (accounting for 95.2% of the total number of publications on agricultural resilience) were used for further bibliometric analysis.

The countries with the highest output of documents on agricultural resilience indexed in Scopus are as follows (**Fig. 1.2**): the Republic of India – 100 documents, the United States of America – 67, the People's Republic of China – 52, the United Kingdom – 39, the Commonwealth of Australia – 25, the Federal Republic of Germany – 25, the Italian Republic – 19, the Kingdom of Saudi Arabia – 17, Canada – 16, the Islamic Republic of Pakistan and the Republic of South Africa – 15 documents each. Ukraine has three indexed documents, accounting for just 0.66% of the global publication output on the subject under study.

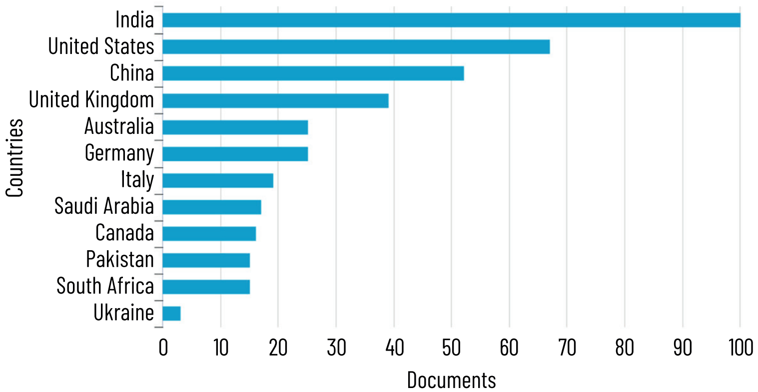

Fig. 1.2 Ranking of the top 10 countries and Ukraine by the number of Scopus-indexed publications containing the terms "resilience" and "agriculture" in their titles

The clustering of cross-country cooperation (**Fig. 1.3**) has identified eight clusters. The first cluster (red) comprises 10 countries: the Kingdom of Belgium, the French Republic, the Federal Republic of Germany, the Republic of Indonesia, the Italian Republic, the Republic of Lithuania, the Kingdom of the Netherlands, the Republic of Poland, the Kingdom of Spain and the Republic of Türkiye; the second cluster (green) comprises eight countries: Canada, the People's Republic of China, the Arab Republic of Egypt, Malaysia, the Islamic Republic of Pakistan, the Kingdom of Saudi Arabia, the Republic of Korea and the Kingdom of Thailand; the third cluster (blue) comprises four countries: the People's Republic of Bangladesh, the Islamic Republic of Iran, Japan and the Swiss Confederation; the fourth cluster (yellow) includes the Federative Republic of Brazil, the Republic of India, the Federal Republic of Nigeria and the United States of America; the fifth cluster (purple) comprises four countries: the Czech Republic, the Republic of Kenya, the Kingdom of Norway and the Kingdom of Sweden; the sixth cluster (light blue) comprises the Commonwealth of Australia, the Federal Democratic Republic of Ethiopia, the Republic of South Africa and the Republic of Zimbabwe; the seventh cluster (orange) comprises three countries – the Republic of Ghana, the United Kingdom and the Socialist Republic of Vietnam; and the final, eighth cluster (brown) comprises the Kingdom of Morocco and the Democratic Socialist Republic of Sri Lanka.

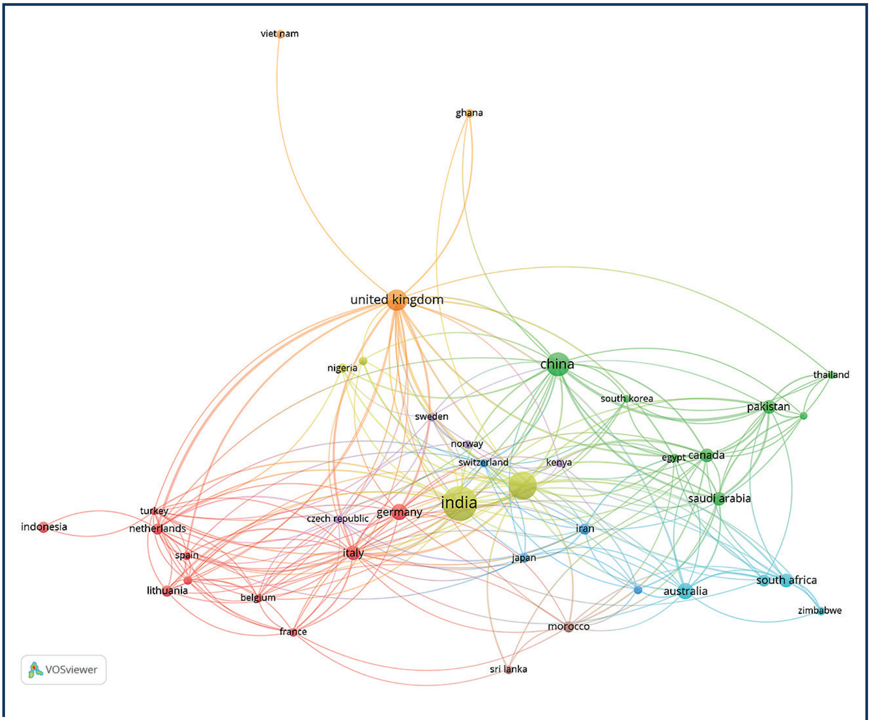

Fig. 1.3 Network visualization of cross-country collaboration in the publication of research on agricultural resilience worldwide

Scientific mapping of cross-country cooperation has clearly identified three key centers of influence with the highest number of connections: the Republic of India acts as a powerful central hub. It has close links with almost all clusters, indicating that Indian science is highly open to international collaboration; the United Kingdom acts as a strategic "bridge" between European countries and the Republic of Ghana and the Socialist Republic of Vietnam. The People's Republic of China dominates the green cluster, focusing on the Asian region and developing countries. Ukraine remains outside the established map of scientific co-operation, which points to the need to step up international research in this area, primarily in collaboration with European research institutions.

An analysis of the leading funding agencies by the number of sponsored research projects and resulting publications reveals that the top five organizations providing the most funding for agricultural resilience research are as follows: the National Natural Science Foundation of China (15 documents), the Horizon 2020 Framework Programme (11), the National Science Foundation (10), the European Commission (8), and UK Research and Innovation (8).

A cluster analysis of publication metadata was carried out based on 145 keywords that appear in the publications at least five times (**Fig. 1.4**). The results of the cluster analysis identified five clusters of interrelated keywords that are most frequently used in publications on agricultural resilience. The first cluster, shown in red, is the largest, comprising 46 keywords. The following terms were most frequently used in the red cluster: agriculture, climate change, resilience, agricultural production, food security. This cluster focuses on the resilience of agriculture to climate change in order to ensure food security.

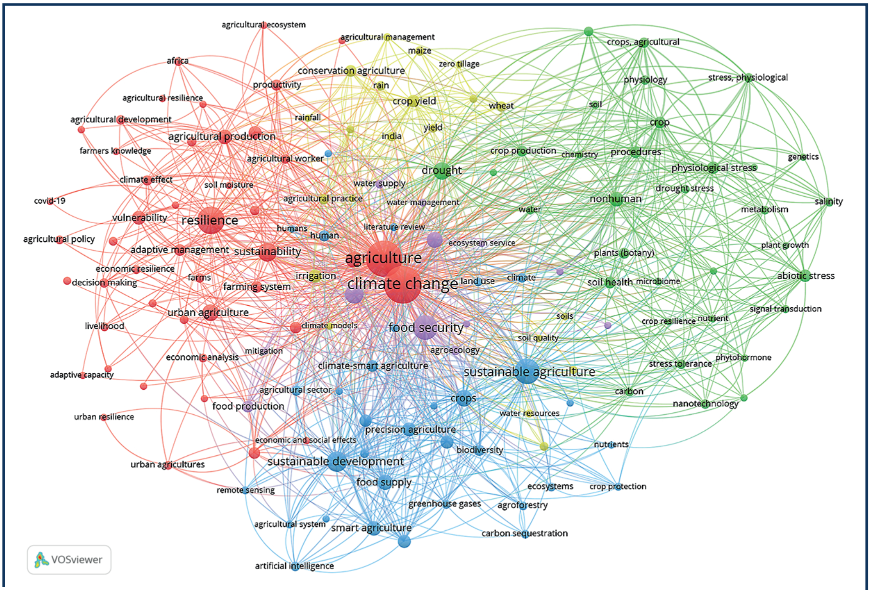

Fig. 1.4 Network visualization of the most frequently used terms in publications on agricultural resilience worldwide

The second cluster, the green one, comprises 33 keywords. The terms most frequently used in the green cluster were: drought, crop, physiological stress, soil health, yield, nanotechnology. The green cluster focuses on biophysical processes, soil and plant systems, their productivity and adaptation to stress factors, including through the application of nanotechnologies. The third cluster, the blue one, also contains 33 keywords. The terms most frequently used in this cluster are: sustainable agriculture, sustainable development, smart agriculture, food supply, and precision agriculture. It focuses on ensuring sustainable agriculture through the implementation of precise, climate-smart technologies.

The fourth cluster, the yellow one, contains 21 keywords. Among the most frequently used terms in the yellow cluster are: conservation agriculture, crop yield, agricultural management, irrigation, and agricultural practice. It covers the study of agronomic practices and agricultural management, in particular irrigation and no-till farming techniques and their impact on crop yields. In fact, this cluster relates to the practical management of agricultural production. Finally, the fifth cluster, the purple one, is the smallest, as it comprises 12 keywords. Among the most frequently used terms are: food security, climate resilience, adaptation, agroecology, and climate adaptation. Thus, this cluster focuses on the adaptation of agricultural production to climate change.

To summarize the cluster analysis, it should be noted that the global scientific community's attention is focused primarily on building the resilience of agriculture to climate change. Whilst considerable attention is paid to crops and soil health, the issue of optimizing crop rotations for sustainable land management and the resilience of agricultural production remains under-researched.

Fig. 1.5 illustrates the evolution of terminology and the development of research in publications on agricultural resilience over recent years.

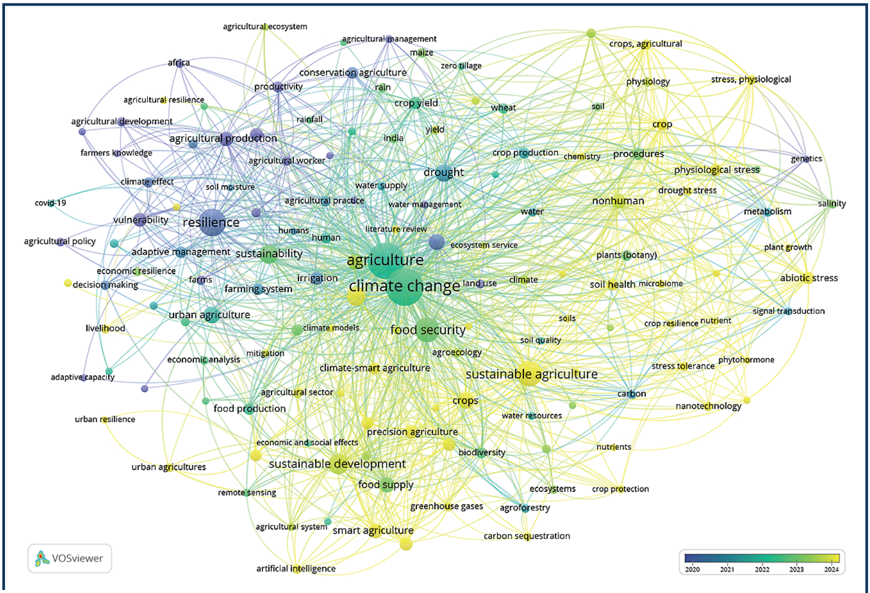

Fig. 1.5 Network visualization of the chronology of term usage in publications on "global agricultural resilience"

In 2020, research focused primarily on the fundamental concepts of resilience and socio-economic aspects in the context of the pandemic. The most frequently used terms in the studies were: resilience, agricultural production, vulnerability, Africa (as a region of active research), and agricultural policy. In 2024, researchers' focus shifted to the latest trends, including digitalization, genetics and deep environmental analytics. Among the most frequently used terms were sustainable agriculture, smart agriculture, artificial intelligence, sustainable development, climate resilience and crops. The visualization shows a clear shift from descriptive vulnerability studies (2020) to technologically complex methods of adaptation and mitigation of climate change impacts (2024), with a particular focus in that year on artificial intelligence and smart agriculture.

The results of an additional combined search (TITLE (resilience AND agriculture) AND TITLE-ABS-KEY ("crop rotation")) among the 436 selected documents identified 16 studies (3.7%) that met the specified search criteria, i.e. they mentioned crop rotation; however, only one document was directly devoted to the issue of applying crop rotation and diversification to improve the sustainability and resilience of agriculture [4]. Scientists note that crop rotation or the introduction of diverse plant species can improve soil fertility, reduce pest populations and increase the availability of nutrients. Furthermore, crop rotation promotes more efficient use of water and nutrients, reducing dependence on synthetic fertilizers and minimizing the risk of pests and diseases [4]. Scientists emphasize the importance of crop rotation as a sustainable agricultural practice; however, there are a number of challenges to its implementation.

A combined search in Scopus using the terms "resilience" and "crop rotation" in the document titles revealed several further documents relating to the subject under investigation. In particular, it was found that increasing the diversity of crop rotation through the use of cover crops enhances the resilience of farms to climate change in the USA [5].

An assessment of the resilience and sustainability of various crop rotations with long-term manure management under future climate change conditions in Canada found that increasing the proportion of perennial crops, particularly legumes, enhanced organic carbon sequestration, whilst diverse annual systems also improved soil health and resilience [6]. It has been established that diversified crop rotations reduce groundwater use and increase system resilience in water-deficient agricultural regions in the People's Republic of China [7]. Although crop rotation can effectively enhance climate resilience and reduce the vulnerability of agricultural cropping systems, farmers often fail to adhere to it in practice; Chinese researchers have therefore proposed:

1) developing crop rotation techniques tailored to local conditions;

2) focusing on the environmental benefits of crop rotation subsidies and the implementation of appropriate and flexible subsidy policies;

3) conducting a rational assessment and adjustment of crop rotation practices [8].

It must be acknowledged that the development and implementation of diversified crop production systems are key factors in the formulation of agricultural policy and a top priority for decision-making at the farm level, with the aim of increasing crop productivity and improving soil health, whilst reducing the negative impact on the environment [9].

Thus, the results of the bibliometric and theoretical analysis of existing studies highlight the benefits of crop rotation and the need to implement it in practice to ensure the resilience and sustainability of agriculture. The first step towards implementing crop rotation is to design it, taking into account existing constraints and the need to optimize resource use.

1.2 FORMULATION OF AN ECONOMIC-MATHEMATICAL MODEL CONCERNING THE PLANNING OF CROP ROTATION USING A COMBINATORIAL APPROACH

One of the key steps towards the effective management of land (soil) resources, which is undoubtedly an integral part of the entire system of soil conservation measures lies in the appropriate crop selection that have a soil-protecting effect against erosion. In this regard, the choice of a methodological approach and, more broadly, an algorithm for optimizing land use is of paramount importance. Central to this approach is crop rotation, which involves a variety of combinatorial arrangements of agricultural crops that must meet a number of conditions shaped by organizational, economic, geodetic, natural, climatic, soil and market constraints.

As a rule, the optimization of on-farm land use and land holdings is carried out according to specific objectives, aiming to maximize the yield of gross (marketable) output (net income) and/or profit. Such criteria are clearly also relevant to an equally specific challenge – the optimization of crop selection when organizing crop rotations, which form the basis of the overall economic structure of land use both at the micro-level (for individual entities and their smallest units) and at the meso- and macro levels, which concerns the scale of regions and the country as a whole.

According to N. Kovalenko, the preference for selecting net profit as the optimization criterion more fully reflects the nature of the optimal solution, as it simultaneously stimulates growth in gross output and reduces running costs, whilst ensuring the full utilization of natural and economic resources and improving soil fertility and overall farming practices [10]. Furthermore, when applying economic and mathematical modelling of anti-erosion processes (measures), the researcher places considerable emphasis on incorporating the crop rotation factor, which has varying degrees of soil protection. She proposes that models for the rational use of land resources in erosion-prone agricultural landscapes should take into account the erosion safety coefficients of agricultural crops [11]. B. Smirnova also emphasizes the significant role of crop rotation in soil erosion control [12].

The primary aim of crop rotation is to achieve consistently high yields, in contrast to the haphazard and unchanging cultivation of crops over time. It is therefore important to rotate crops and to ensure they are returned to the same field at regular intervals as part of the crop rotation system [13]. Crop rotation and the proper balancing of agricultural sectors contribute to making farming more environmentally friendly; in particular, a humus surplus of 0.33 t/ha or more is achieved when applying an objective function aimed at maximizing economic efficiency (net income/profit) [14]. The humus balance across crop rotations was calculated using a methodological approach that accounts for sources of humus input and losses, including those from by-products.

The use of an objective function aimed at maximizing revenue or profit – considering only resource utilization while excluding environmental constraints is a traditional approach to finding an optimal solution in an economic-mathematical model. One option for optimizing crop rotations is linear programming, which is predominantly based on the simplex method, which many sources suggest using to find the optimal set of values for unknown variables [10–15]. However, the vast majority of them already incorporate an environmental component, which establishes an eco-economic focus for optimization within land-use structures.

Alongside linear models, dynamic programming (modelling) is frequently employed, which in this case acts as an independent optimization method, successfully used for the optimal selection of crops and the effective organization of crop rotations [16].

However, the economic-mathematical models outlined above lack the capacity to fully explore all possible options in the search for optimal solutions. This necessitates combinatorial arrangements with combinatorial search methods, including graph-based optimization techniques. Consequently, this gives rise to a specific class of economic models that should be applied subject to certain conditions.

In general, formulating a combinatorial problem involves searching for and reviewing various combinations of elements. When designing crop rotations, there is a need to repeat crops previously used in a rotation sequence in subsequent stages, that is, in the long term over the coming years. This necessitates the development of models incorporating combinatorial repetition. However, not all classes of combinatorial problems involving the formulation of crop rotations are suitable for this purpose, because in crop rotations the order of numerical permutations is significant; the combination of two crops, depending on which is the preceding crop and which is the subsequent one, is entirely different in terms of biological and agro-physical conditions. In other words, such combinations are distinct, and for these two crops, the possible combinations should be considered as two separate cases.

In this situation, it is necessary to rationally select a sequence of crops in order to establish a limited number of crop rotations capable of utilising soil (land) resources most effectively. Reviewing all options for a combinatorial solution enables the identification of the global optimum and fully satisfy the initial conditions regarding the specified optimization criteria (organizational, technological, natural and climatic, soil-related, etc.).

Problems concerning the solution of programming problems based on combinatorial optimization, where a number of constraints on the elements of combinatorial sets are taken into account simultaneously, have been addressed in various studies [16–18]. In turn, important issues concerning the functioning of the apparatus for implementing mathematical models of combinatorial optimization problems are discussed in the scientific publication [19].

The overall structure of the optimization model takes into account key vectors that describe a range of fundamental indicators of the functioning of biological and economic systems, without considering environmental constraints. In other words, it is not possible to take into account the environmental component, which would impose clear restrictions on the average use of land (arable land) for field agriculture depending on soil erosion control and the preservation of sustainable land use, in particular the regeneration of the soil organic matter.

Thus, the task statement concerns the study of optimization of crop rotations and their organization, taking into account the fundamental requirements for crop succession, the selection of preceding crops, and the maximization of financial returns (profit) per hectare of such rotation area or across the entire rotation. Accordingly, optimal crop combinations must be identified (notwithstanding their potential triviality in certain cases), and various methodologies should be examined to complement mathematical approaches in seeking optimal solutions for managing complex agricultural production and land-use systems; this includes the graphical representation of resulting elements to ultimately identify the optimal set of indicators and determine the optimal allocation of crop areas while assessing erosion risk within the framework of this general model.

Considering many factors in combinatorial optimization is known to give rise to the "curse of dimensionality"; consequently, even when searching for the global optimum (extremum), it is possible to fail to complete the organized process at all, having first run a computer algorithm based on a mathematical model. For input data of small dimensions in combinatorial optimization, calculations can be performed using an exhaustive search method; otherwise, search-based optimization methods are used, since when searching for the extremum of a function, the number of combinatorial configurations grows exponentially depending on the elements of the combinatorial sets (fields, crops, etc.). In studies involving an exhaustive search of combinatorial arrangements, mathematical models have been developed; however, these do not resolve the issues relating to dimensionality [20]. Furthermore, the determination of the size of a combinatorial set is limited by the capabilities of computing technology. In particular, on a personal computer, optimization problems involving an exhaustive search are limited to 13 elements of combinatorial configurations; that is, the total number of fields is equal to the number of crops, or $k = 13$ units. Conversely, the use of powerful computers makes it possible to expand the scope of element enumeration; however, the set of elements required for a complete enumeration amounts to approximately 20 combinatorial arrangements, and therefore it is recommended to solve problems of this magnitude $k > 19$ using search-based optimization methods [18].

The time required to solve a combinatorial problem, which is related to the choice of an appropriate method, increases in direct proportion to the number of elements in the set of combinatorial arrangements; however, if no constraints are imposed on the set of elements in the mathematical model, the total time required to find a solution is reduced, since one of the three stages is eliminated, which would otherwise have to verify compliance with the constraints on the solutions obtained during modelling $\left(T_2(k) = k \cdot t_2 = 0\right)$. Thus, the time required to find a solution will be $T(k) = T_1(k) + T_3(k) = k\left(t_1 + t_3\right)$, that is, it consists of the time taken to obtain a combinatorial arrangement by exhaustively enumerating the set of elements (generating cases) $T_1(k)$ and includes the calculations required to determine the objective function of the problem for all possible combinatorial arrangements obtained for a specific mathematical model (combinatorial problem) $T_3(k)$ [19]. Consequently, the reduction in the time required to find a solution to a problem is not solely due to a decrease in the number of elements in the set of combinatorial arrangements. The presence of constraints in the mathematical model and the complexity of constructing its the objective function also significantly influence this parameter; ultimately, these factors dictate the choice of the optimization search method.

Notably, when dealing with a large number of elements k and a complex mathematical model in terms of the constraints imposed upon it, search methods can be combined and/or supplemented. It should be borne in mind that the optimization criterion $W\left(Z\right)\left[r, u^*, I\left(r, u^*\right)\right]$ (equation 1.1) for a given system in the problem is non-linear in nature. In this case, to find the optimum, it is necessary to apply not all numerical optimization methods, but only those based on multidimensional approaches to non-linear programming [21]. The use of a randomized method, in view of the parametric uncertainty of the input data, ensures an effective and robust estimation of the sought-after quantities, solving test problems in both deterministic and stochastic forms of description (formulation) [22].

When considering a specific set of elements at various levels of coordination for the tasks involved in the optimal organization of crop rotations, some of these problems are solved using modern methods of analysis and the generation of numerical sequences, in particular the Monte Carlo method. As V. Putyatyn and S. Kovalenko argue, if the number of crops n is equal to (identical to) the number of fields m, then the number of elements in the combinatorial set will be $n!$ of permutations. However, obtaining additional solutions based on computational experiments using simulation methods only partially improves the situation, and the problem of dimensionality cannot be avoided; therefore, to solve the combinatorial problem, researchers propose simultaneously applying a number of supplementary methods, namely the Monte Carlo method, the Neighborhood Reduction method and the Descent Vector method [17].

The combinatorial arrangement of crops across sequences allows for the formulation of a crop rotation model designed to maximize economic returns while satisfying agrotechnical and agroecological constraints; the search for the optimal value is assessed using the criterion r^* (1.1), (1.2). The basic form of constructing and recording the mathematical model has been developed in the scientific works of V. Putyatin and S. Kovalenko [17, 18].

$$\begin{gathered} extrW\left(Z\right)\left[r,u^*,I\left(r,u^*\right)\right], \\ r\in R^*\subset R; \end{gathered} \tag{1.1}$$

$$\begin{gathered} r^*=\arg extrW\left(Z\right)\left[r,u^*,I\left(r,u^*\right)\right], \\ r\in R^*\subset R, \end{gathered} \tag{1.2}$$

where r^* – the optimal value of an element of a combinatorial set; $W(Z)$ – criteria for optimizing the selection of agricultural crops and/or the sequence of crops over time (with W or Z chosen approach, if the dual nature of their independent search is conventionally considered); r – a combinatorial set based on the input values of agricultural crops; I – a vector of its own criteria set; u^* – the optimality vector for the given constraints in the model when formulating combinatorial arrangements within the framework of requirements, including environmental ones, for the formulation of crop rotations.

The general output of this model is represented as a corresponding tuple $\left\langle r^*,W^*\left(Z^*\right),I^*,u^*\right\rangle$, which is obtained through combinatorial computation, where crops are selected subject to environmental constraints regarding the properties of preceding crops in the formation of an optimal crop rotation (though not regarding the use of soil resources).

Let's consider a model in which the number of crops k is greater than the number of fields (plots of land available for allocation) s and/or the number of possible allocation scenarios over time t, i.e. when $k > s$ or $k > t$, but $s = t$, and in this case it is not possible to consider possible options for forming composite fields, which constitute the corresponding combinations on a subset, where for the latter the set A will already have a subset 2^A, when $A\in F_n$ or from the entire set according to the option in the graph representation [23]. Furthermore, this scenario is described quite clearly in the well-known Erdős-Rényi model, which combines combinatorial arrangements with the probabilistic occurrence of an event leading to the generation of random graphs [23], including the emergence of subgraphs.

According to the definition of a combinatorial problem, the elements of its set are those that satisfy condition (1.3):

$$r=\left(i_1,i_2,\ldots,i_k\right);\, i_{w(z)}\neq i_r \text{ for } w(z)\neq r, \text{ when } i_{w(z)}\in j_{s(t)}, \text{ but } w(z),r\in j_k, \tag{1.3}$$

where the j – field with $\{S_{1,n}\}$ – vector-row or over time $\{T_{n,1}\}$ – a column vector for the i crop (i_j).

Another essential condition is the imposition of a prior restriction on combinatorial arrangements that are inadmissible under the requirements of the crop rotation system concerning the sequence of preceding crops, where $r_f\left(f=1,2,\ldots,\mu\right)$ are excluded from R, restriction (1.4). One of the important constraints is the issue of crop repetition as a preceding crop to prevent soil exhaustion and ensure yield stability; in this case with R, it is necessary to prevent by the list (scope) to $r_g\,(g=1,2,\ldots,t)$, condition (1.5):

$$r\neq r_f\,\left(f=1,2,\ldots\mu\right); \tag{1.4}$$

$$r\neq r_g\left(g=1,2,\ldots,t\right). \tag{1.5}$$

In this context, it is important to impose a restriction whereby a single crop is assigned to a single field (1.6) and, as an additional condition, to introduce a criterion for evaluating the achievement of the selection (1.7):

$$i_{w(z)}\neq i_r \text{ for } \forall\;\; w(z)\neq r\,\left(w(z)=1,2,\ldots,s(t)\right); \tag{1.6}$$

$$r>r_e\left(e=1,2,\ldots,c\right) \text{ if } \omega(r)>\omega(r_e), \tag{1.7}$$

where the correspondence from e technological by c set of methods for i agricultural crops, or the economic efficiency of an agrotechnical or other practice, is prioritized in accordance with the ω selection evaluation criteria.

The search can be terminated by imposing acceptable (satisfactory) constraints on economic feasibility (1.8):

$$\begin{aligned}&\left|\varphi(r**)-\varphi(r*)\right|\leq\varepsilon;\\&\quad R**\subset R*\subset R,\end{aligned} \tag{1.8}$$

where φ – the criterion of the expected value obtained using a computational or expert approach for a given (desired) combinatorial arrangement (r**), where the optimal solution corresponds to the option in which the criterion r* will satisfy the specified value without exceeding the established limit (standard) ε.

However, the constraints considered eliminate the need to exhaustively search through all possible combinatorial arrangements arising from the set of crops to be scheduled over time. On the other hand, this eliminates the problem of generating subsets, as the application of the graph approach will prevent the emergence of graphs (subgraphs) and the risk of cyclic connections between their vertices, since a cycle is a subgraph of graph *G* consisting of the vertices and edges through which it passes. That is, the induced cycles of a graph are those cycles that act as its induced subgraphs [24]. According to Dirac's theorem, a graph in which the vertices have degree $\geq n/2$, is characterized by cyclicity, without reference to the cyclicity of a path in which there is repetition along the edges of the graph [24]. Furthermore, it is known that if at least one edge is added to an acyclic (cycle-free) graph, then a single cycle will appear in it [24].

The search for multiple solutions to a crop rotation problem involving cycles significantly complicates short-path algorithms and, in some cases, leads to premature intervention and programme termination. Such cases should be avoided, which will allow for more effective use of standard algorithms, in particular Dijkstra's algorithm [25]. It should be noted, however, that not all algorithms are capable of handling negative edge weights (arcs), as one of the key components of crop rotation is economic performance; in some cases, a loss is permitted for certain crops, even though they are the best preceding crop for high-yielding crops in the rotation. However, such difficulties can be overcome using other methods (algorithms), in particular by incorporating interval parameters or sub-objectives within the overall optimization of crop allocation in the crop rotation [26]. One such example is the analysis of crop rotation planning (organization) on a hypergraph, achieved by finding the optimal vertex coverage in a hypergraph with fuzzy weights constructed according to the problem's conditions [27]. Also worthy of attention is the consideration of models which, with regard to the search for optimal solutions, involve forecasting time series based on the application of cellular automata and fuzzy theory [28].

As N. Tymofieva notes, in combinatorial optimization, the objective function depends on both a single variable and several variables, which are combinations of various types, whereas the variables in an integer linear programming model serve as input data. The researcher therefore proposes dividing them into several sub-problems, using independent algorithms sequentially or in an iterative manner [29]. Despite the need to identify sub-tasks, it is necessary to clarify the conditions for consistency of set dimensions.

The first sub-problem involves selecting crops for each field, i.e. permutations of *m* crops from the elements $m=\overline{1,\ldots,l}$ and the corresponding coverage (allocation) of the field for *n* fields from the elements $n=\overline{1,\ldots,i}$, which forms the set of permutations *Ω*, to which the elements of the set ω already belong, and where the latter acts as a transition to the subsequent sub-problem in the combinatorial arrangement; in particular, it involves selecting the permutations already obtained according to a set of climatic and weather conditions from the set *P* and thereby forming a set of arrangements without repetition *M* ($\mu \in M$) [29].

Thus, the function for the task is written as $F\left(\omega^{*},\mu^{*}\right)=\underset{\substack{\omega\in\Omega\\ \mu\in M}}{extr}F\left(\omega,\mu\right)$, however, the conditions for solving the combinatorial problem using permutations for the first sub-problem and arrangements for the second are not specified [29]. At the same time, it is clear that, since $C_{\omega}^{\mu}\begin{pmatrix}\omega\\ \mu\end{pmatrix}$ if $\mu \succ \omega$ is equal to zero, then a set comprising the climatic and weather conditions *M* across the entire set of permutations *Ω*, which is attainable only if $\mu \leq \omega$, and these permutations, in turn, satisfy the condition that the elements of *l* = *i* correspond to those of the set $|A|=|B|$ (the cardinality of the sets), where $A=\{m_1,\ldots,m_l\}$ and $B=\{n\ ,\ldots,n_i\}$. In other words, the mathematical model of the basic combinatorial optimization problem involves a permutation *m* crops on fields, when *m* = *n* [26].

In this case, the search for the optimal combinatorial arrangement is restricted to the permutation with the greatest number of fields that are identical to those selected for consideration across all crop variants; alternatively, in the search for the global optimum, this acts as an upper bound determined by the crop with the highest ordinal number from the entire set *A*.

In this case, there are certain discrepancies in the technical execution of the calculations, as the basic combinatorial formulas may be applied differently in the various options; however, it should be noted that they are used in accordance with the rules of combinatorial problems only after the general framework of the investigation has been established. Here, it is necessary to clearly formalize the situation with the subsequent search for a set of solutions.

Optimizing a crop rotation system requires accounting for key factors during calculation; analysis of the results obtained comparing the final solution to the optimization problem with an indicative numerical set already derived from the combinatorial arrangement of possible variants based on the given initial set size of the combinatorial problem. Assuming it is possible to select from a set of crops $A=\{m_1,\ldots,m_l\}$ to allocate particular crop to a specific field from a given set of fields, $B=\{n_1,\ldots,n_i\}$; then it is clear that the ordering of the set of fields, which is smaller than the required set of crops $(|A|>|B|)$, reduces to ordering these crops, no longer according to the permutation scheme $(P_m>P_n)$. However, in this case, when calculating the permutations of crop arrangements, this occurs as a special case of formulating a combinatorial problem.

A key requirement for combinatorial optimization of crop rotations is adherence to the sequence of crops within each rotation. Previous crops are classified as unsuitable, conditionally suitable, suitable and optimal. The crop combination is, of course, significant, as an inappropriate choice of previous crop will negatively affect the yield potential of the subsequent crop. Thus, the combination of winter wheat and grain maize is not the same as the reverse combination. The first case represents the optimal match between the preceding crop and the subsequent crop, whilst the second, conversely, is unacceptable.

Consequently, the order of an element and/or its position in the sequence is of great importance, as this determines whether the permutations are of a completely different nature or even inadmissible. Another aspect is the different set of elements between the set for ordering crops (*A*) and the order of their allocation in a specific field (*B*); it is also permissible for a crop to be repeated if it alternates in sequence on the same field. For instance, crops such as perennial grasses, winter wheat, or maize can be replanted in the same field consecutively, but must be followed by a crop rotation break.

Therefore, when arranging crops across fields, there are two possible ways of applying the variation formula: without repetitions and with repetitions. The latter offers a wider range of possible combinations and, in this case, becomes a standard optimization problem.

Let's consider a dynamic version of crop rotation, in which there is a set of agricultural crops A_i, consisting of elements $i=\overline{1,\ldots,n}$ and their allocation over time, which is defined by the set T_j, where $j=\overline{1,\ldots,t}$. In this case, the condition on the sets satisfies the requirement $|A_i|>|T_j|$, or the number of possible combinatorial arrangements of crop allocations is limited to the size of the *j*-time slot from the general group (set) *T*, which is precisely the value of *t*.

Crop allocation over time (with or without repetitions) is expressed by formulas (1.9) and (1.10):

$$\overline{A_n^t} = n^t; \tag{1.9}$$

$$A_n^t = \frac{n!}{(n-t)!}, \tag{1.10}$$

where n – agricultural crop; t – allocation over time; $n!$ – permutation of crops regardless of time constraints (representing all possible ordering variants), where the factorial is the product of all elements in the sequence and follows the rule 0! = 1.

In cases where the number of agricultural crops equals the time duration (the dimension of the crop rotation), this arrangement simplifies to a standard permutation (1.11)

$$A_n^{t=n} = P_n = n!. \tag{1.11}$$

It should be noted that, unlike the previous condition ($n = t$), in this case an arrangement with repetition does not in any way become a permutation, as these are entirely different schemes of combinatorial arrangements, differing in the composition of the groups and the very formulation of the combinatorial problem.

Given the nature of the dynamic combinatorial problem, it is necessary to break down the element t into its constituent dependencies. Let's assume that a change in the time step, which corresponds to one year of cultivation of the i crop, can be defined as a change in the permutations of these elements over the corresponding periods (1.12)

$$t = \frac{P_t}{P_{t-1}}, \tag{1.12}$$

where P_t – the number of permutations of elements from a set T_j for t year; P_{t-1} – the number of permutations of elements in a set T_j for t–1^{st} year.

The permutation of elements in the t year can be represented as a set of arrangements and combinations without repetition for agricultural crops, with the set having a cardinality of $|A_i| = n$ (1.13)

$$P_t = \frac{A_n^t}{C_n^t} = \Gamma\left(t+1\right), \tag{1.13}$$

where C_n^t – arrangements without repetition of the n crop rotation; Γ – the gamma function for a numerical value t+1.

In other words, the permutation of elements by year corresponds directly to the variations without repetition by the i crops, but has an inverse relationship with their combination. On the other hand, the variation without repetitions $\left(A_n^t\right)$ is the product of all permutations of elements in the dynamic (time) sequence over the t year of the crop rotation and the combinations of i agricultural crops from the j time interval or from n elements of the set over t $\left(C_n^t\right)$. When shifting the time by one step in a permutation operation, it is also necessary to multiply by the crop rotation size (1.14). The latter value can be obtained by combining the relevant gamma functions (1.15) or using combinatorial formulas (1.16):

$$A_n^t = t \cdot C_n^t \cdot P_{t-1}; \tag{1.14}$$

$$t = \frac{\Gamma(t+1)}{\Gamma(t)}; \tag{1.15}$$

$$t = \frac{A_n^t}{C_n^t \cdot P_{t-1}}. \tag{1.16}$$

Substituting the crop rotation index or parameter *t* (1.16) into the combinatorial variation formula with repetitions (1.9), it is possible to obtain a direct relationship between the latter and the crop allocation index, but without repetitions and with corresponding gamma functions of numerical values in accordance with the model parameters (1.17). Ultimately, the crop rotation formula (1.18), derived from analytical expression (1.16), is transformed into a ratio of the variation and combination of set elements for the *t* time period, divided by the gamma function of the size of the crop rotation itself or for the same time period:

$$\overline{A_n^t} = n^{\left[\frac{A_n^t}{C_n^t \cdot P_{t-1}}\right]} = n^{\frac{\tilde{A}(t+1)}{\tilde{A}(n-t+1)}\left[\frac{(n-t)!}{(t-1)!}\right]} = n^{A_n^t\left[\frac{\tilde{A}(n-t+1)}{\tilde{A}(t)}\right]}; \tag{1.17}$$

$$\overline{A_n^t} = n^{\frac{1}{\tilde{A}(t)}\left[\frac{A_n^t}{C_n^t}\right]}. \tag{1.18}$$

Therefore, the allocation of crops with repetitions over time $\left(\overline{A_n^t}\right)$ depends on the number of crops (*n*), raised to the power of the number of distinct combinatorial arrangements of their allocation without repetitions $\left(A_n^t\right)$, and on the product of the ratios of the corresponding gamma functions. Thus, one of these represents the difference arising between the set of crops and the years of their allocation in the crop rotation, whilst the other is the value of the crop rotation itself over *t* years.

The gamma functions under consideration (1.17) can be expressed in factorial form, which ultimately takes the simplified form (1.19)

$$\overline{A_n^t} = n^{A_n^t\left[\frac{(n-t)!}{(t-1)!}\right]}. \tag{1.19}$$

After determining the crop rotation dimension for the *t* period, the formula for crop allocation without repetition takes the following form

$$A_n^t = \frac{\Gamma(t+1)}{\Gamma(n-t+1)} = t \cdot \frac{\Gamma(t)}{\Gamma(n-t+1)} = t \cdot C_n^t \cdot (t-1)!. \tag{1.20}$$

A comparison of crop allocation with and without repetitions within a crop rotation over *t* years is presented analytically (1.21) through substitution and transformation of previous expressions.

The value significantly depends on the root degree, as the radicand grows rapidly, which, in turn, establishes a corresponding link between the two main indicators of combinatorial calculation

$$\overline{A_n^t} = \sqrt[\left[\frac{\tilde{A}(n)}{\tilde{A}(n-t+1)}\right]]{n^{A_n^t}} = \sqrt[\left[C_n^t \cdot P_{t-1}\right]]{n^{A_n^t}}. \tag{1.21}$$

It should be noted that the radicand is greater than zero, however, for real numbers, the condition $n \geq t$ must hold. Therefore, it follows from equality (1.21) that the numerical predominance of combinatorial arrangements from the set of agricultural crops with repetitions over cases where such repetitions of crops did not occur in crop rotation sequences was previously shown in an obvious but not combinatorial manner. For example, given a fixed number of crops n = 10 and t = 3 years, the ratio of crop allocation schemes involving repetition to those without repetition $\left(\overline{A_n^t} / A_n^t\right)$ is (1.4), and the degree of the root is 90. If in the crop rotation cycle t equals 5 and 7 years, the ratio of combinatorial arrangements increases up to 3.3 and 16.5 times respectively, then the degree of the root rises sharply to 5,040 and 151,200. It follows that: since $n^{A_n^t} >> \dfrac{\Gamma(n)}{\Gamma(n-t+1)}$, the magnitude of the indices corresponding to the placement of crops will be similar, i.e. $\overline{A_n^t} >> A_n^t$.

1.3 ADDRESSING CROP ROTATION CHALLENGES USING THE TRADITIONAL APPROACH CONSIDERING INSUFFICIENT ENVIRONMENTAL CONSTRAINTS

The broad scope of the search for optimal solutions when designing crop rotations is based on identifying the best combinations of crops using various methods; in particular, it is quite common to locate critical points in space when the scalar of the sought variables is multidimensional. Numerous discrete methods facilitate this. In a study on the Crop Rotation Problem (CRP), B. Santos proposed a model in which the input parameters are an empty set of plots, as well as the total number of periods and a set of crops [30].

This approach to finding the optimal set of parameters uses linear programming in a multidimensional space. At the same time, binary parameters were used in the constraint equations of the proposed model. This choice specifically concerned the unknown variable, which represents a particular plot in the crop rotation, where the condition takes the value 1 when fulfilled, and 0 otherwise, i.e. is rejected.

In this context, L. Santos et al. [31] proposed an optimization model in which a binary approach to crop rotation conditions allows the placement pattern for each plot within the sowing area to be determined. Crop rotations have the same duration across all plots, and crops are selected to ensure the plot is utilized as fully as possible. Researchers note that, in general, there is a problem with incorporating cropping restrictions for neighboring plots, as well as for the sequence of crops in the crop rotation. Therefore, to simplify and/or facilitate the solution of the optimization problem, modifications have been made to the developed model based on a heuristic approach and the generation of input data.

Researchers A. Aliano et al. [32] proposed a mathematical model to solve the crop rotation problem with demand constraints, or CRP-D (Crop Rotation Problem with Demand Constraints); this problem was solved using a developed genetic algorithm, including simulated annealing and hybrid approaches. In other words, the search for optimal solutions to crop rotation problems was based on heuristics, which, in turn, provided initial solutions for meta-heuristics.

In addition, researchers B. Miranda et al. [33], considering the need to transform the technological framework of agriculture 4.0, proposed a new mathematical approach to solving the problem of crop rotation, which would meet farmers' expectations, including by taking into account the numerous resource constraints and requirements involved in planning crop yields. All of this is based on the development of an optimization model using a genetic algorithm. Meanwhile, A. Aliano et al. [34] attempted to solve the crop rotation problem using a dual approach, in contrast to the economic-mathematical models considered previously. In the course of solving the dual problem, it was found that the profitability and diversity of crop rotation are conflicting objectives; however, the proposed approach achieves a degree of reconciliation between these diametrically opposed goals.

In this regard, A. Mehrabian and C. Lucas [35] propose applying the Invasive Weed Optimization (IWO) algorithm, which they consider one of the most effective numerical methods for stochastic optimization. This approach has proven to be superior to other well-known algorithms, such as Genetic, Memetic, Particle Swarm, and Shuffled Frog-Leaping algorithms.

Researchers R. Forrester and M. Rodriguez [36] have developed a mixed-integer programming model to address crop rotation challenges. Their approach focuses on optimizing crop sequences within a four-year cycle on an organic farm, accounting for weed control, soil nutrient depletion, irrigation methods, and market demand. With regard to the objective function, it should be noted that the authors have chosen to optimize not by maximizing profit, but on the basis of forecast demand for the organic farm's produce. Consequently, the model includes four main indices, namely:

1) crops;
2) fields;
3) years;
4) months.

The crop cultivation optimization criterion aims to reduce unmet demand based on the forecast value.

Regarding the aforementioned model, it is both traditional and innovative. It redefines the objective function and the set of constraints by incorporating a proportionality coefficient for crop group selection. Furthermore, the environmental orientation is enhanced through the spatial diversity and differentiation of crops. Furthermore, J. Fendji et al. [37] developed a linear programming-based crop rotation optimization model for an organic farm, where the objective function is the farmer's income and which incorporates three levels of constraints:

1) biophysical, relating to crop sequence and plot adjacency;
2) structural, which include the producer's budget and resources;
3) organizational, which incorporate nutrient amendments and reflect market demand.

W. Haneveld and A. Stegeman [38] proposed a crop rotation model using traditional linear programming for multi-year agricultural planning. By applying the traditional method of linear programming to multi-year agricultural production planning, they include crop restrictions based on preceding crops, thereby demonstrating that, under certain conditions of regularity, a stationary cropping plan is the optimal solution for such a model.

Researchers K. Deep et al. [39] employ a mixed-integer linear model to address the problem of crop rotation, which incorporates a genetic algorithm to identify the optimal set of solutions. Furthermore, the objective function of this mathematical model is to maximize the net profit derived from the crop rotation plan. Overall, adherence to the principles of crop rotation optimization and design is a key factor in improving various outcomes in agriculture, and advanced decision-support systems for crop rotation in space and time offer significant potential for enhancing the resilience of the food system [40].

One of the major challenges in designing crop rotations and, consequently, in optimizing them through an economic-mathematical formulation of the problem is addressing the issue of separating crops from neighboring and/or adjacent plots. This problem arises due to the varying ability of crops to resist various adverse factors, in particular the development of pests, plant diseases, weed infestation, etc. In a study conducted by G. Mauri [41], a mathematical model was developed which takes into account the botanical characteristics of different crop families when designing their planting patterns and improves the purity of the crop (enhancing phytosanitary conditions), which has a beneficial effect on adjacent plots. At the same time, thanks to the correct organization of crop rotations, advantages are created in counteracting changes caused by the physical, chemical and biological properties of the soil over time. In particular, the number of pests is reduced, the availability of nutrients is improved, and economic and climatic risks are reduced [42].

In the mathematical model by L. Santos et al. [43], the crop rotation problem is framed in terms of the need to maximize the utilization of land plots with identical yields and climatic conditions; plot sizes remained constant, but certain constraints applied to crop placement, stipulating that crops belonging to the same botanical family could not be grown simultaneously on adjacent plots. The aim of the optimization was to develop a crop rotation that would increase profits whilst meeting the known demand for produce.

Researchers S. Li et al. [44] proposed an agent-based model that already considered possible scenarios for change, particularly climate-related ones, although it is based on numerous forecasts and assumptions. At the same time, M. Kang and F.-Y. Wang proposed a transition from parallel to intelligent crop management based on the application of artificial intelligence, combining a sequence of three steps: yield modelling, forecasting and allocation [45].

A new methodological approach to solving the crop rotation problem was proposed by L. Santos et al. [46], who developed a crop rotation concept based on the linear generation of mixed integers, which involves linear constraints regarding compliance with environmental production conditions and constitutes a single-objective problem. Its aim is to achieve optimal coverage of demand for the products produced. The solution technique involves various methods (column generation by indicators, the use of heuristic methods and estimates).

Researchers A. Ridier et al. [47] have examined the issue of transitioning to long-term crop rotations, which they consider to be innovative. As a result of research into the optimization of crop allocation, the researchers identified the impact of production and market risks on the final decision-making regarding the selection and/or design of crop rotations with an expected duration of up to 25 years. According to the mathematical model, production risk is determined by intra-year constraints, whilst market risk is determined by inter-year constraints; however, the latter has a significant impact on future crop yields. This model is based on discrete stochastic programming and sets the objective of maximizing gross profit, i.e. it takes into account the optimization of crop rotation in light of agronomic and economic conditions [48].

The heterogeneity of the data sample for econometric models is of significant importance when allocating resources and optimizing crop placement; therefore, this issue must be addressed as a matter of priority. One possible approach is to assess the empirical relevance of theoretically defined problems [49]. Mathematical models for optimizing cropland area and crop placement can generally be classified as decision-support models, which involve two key concepts: the cropping plan and the decision regarding the crop rotation itself. However, this decision has a significant impact on resource efficiency, the environmental sustainability of agricultural production, and the landscape [50].

There are still methodological challenges involved in modelling real-world processes in crop rotations. These are linked to the need to account for a broader spectrum of effects in accordance with life cycle assessment (LCA), i.e. the integration of the crop rotation system into the LCA, including all interactions between individual crops in the rotation [51]. A highly effective tool for crop optimization is the use of high-resolution historical crop maps. Thanks to deep neural networks, these maps enable the creation of pre-season crop type maps, which serve as predictive data for crop rotation, analyzing them both spatially and temporally [52]. This approach enables crop type prediction for each field prior to the harvest season with approximately 60% accuracy, outperforming existing methods based on remote sensing [53]. Furthermore, a simple neural network architecture has been developed to simultaneously model intra-annual and inter-annual agricultural dynamics. This modification achieves high precision, reducing the error rate by over 21% compared to state-of-the-art crop classification methods [54].

Generally, the problem of crop rotation design comprises two key aspects: the first relates to the organization of the rotation pattern, and the second to the selection and management of the implementation and establishment of the crop rotation. At the same time, the optimization of crop selection takes into account a number of necessary constraints, which allows sufficient efficiency in achieving the set objectives even with linear integer programming. However, the application of modern algorithms and techniques leads to a variety of solutions, increases the accuracy of estimates and the flexibility of the developed crop rotations to natural and market conditions.

To determine the combinatorial arrangements of crops within the model, the simplex method is applied to maximize profit per rotation. This is subject to basic-level constraints that enforce temporal sequence rules for the crops. In other words, this model develops a dynamic crop rotation and takes into account the characteristics of preceding crops, whilst one of the factors of sustainable land (soil) use is the preservation of humus per rotation, which will be replenished by root systems, crop residues and by-products from agricultural crops. In other words, the model crop rotation is market-oriented with minimal compliance with environmental requirements, but adheres to the general principles of crop rotation planning. The key consideration here is to analyze the marginal impact of crops when designing the field crop rotation.

The condition for maximizing profit over the entire crop rotation period is as follows (1.22):

$$F_c = \operatorname{argmax} \sum_{r=1}^{d} \sum_{i=1}^{n} \sum_{j=1}^{t} \left[\left(W_i \cdot Y_{ri} \cdot U_{ij} \cdot P_i - C_i \right) \cdot X_{rij} \right], \tag{1.22}$$

where F_c – the profit for the crop rotation cycle of the *t* period, UAH;

W_i – an estimated (projected) yield of the *i* crop, q(cwt)/ha;

Y_{ri} – the coefficient of dependence of the *i* crop on the *r* preceding crop;

U_{ij} – the binary number corresponding to the standard rotation period for the *i* crop in the *j* year of the crop rotation. This parameter is defined as $U \in \{0,1\}$;

P_i – an accepted (calculated) price of the *i* crop, UAH/q(cwt);

C_i – the standard (normative) costs for a particular cultivation method for a *i* crop, calculated per 1 ha of sown area, UAH;

X_{rij} – planting a plot (field) with a *i* crop in the *j* year of the crop rotation following the *r* preceding crop, calculated per unit area, ha.

The magnitude of the response to the filling of a plot is determined using equation (1.23), and the indices used in it are defined using equation (1.24):

$$X_{rij} = \begin{cases} 1, \text{if } i \text{ crop after } r \text{ preceding crop in the } j \text{ year;} \\ 0, \text{otherwise,} \end{cases} \tag{1.23}$$

$$X_{rij} \in \{0,1\}, r \in I_i, i = \overline{1,\ldots,n}, j \in I_i. \tag{1.24}$$

The restriction on growing more than one of the *i* crops in the *j* year, i.e. the spatial and/or temporal prevention of the simultaneous occurrence of two or more crops on the set in question, is defined by the inequality (1.25)

$$\sum_{i=1}^{n} \sum_{h_i=0}^{j_i-1} X_{ri(j-h_i)} \le 1, \tag{1.25}$$

where h_i – a possible change in the number of *i* crops, allocated in space or time for the *j* period;

j_i – the suitability of the *i* crop allocation in rotation for the *j* period, where it is stipulated that $j \le i$;

I_j – the period of the *j* year in which it is permitted to allocate (sow) the crop from the *I* set.

Compliance with the restriction condition specified in inequality (1.25) serves as a safeguard against the repetition of crops in the same year or on the same field. This is achieved if $X_{rij} = 1$; in turn, this requires that $X_{ri(j-h_i)} = 1$. This is possible when $h_i = 0$ and confirms the absence of repeat allocation of the *i* crop in the *j* period or, more generally, from $j - (j_i - 1)$ to the *j* period of the crop rotation.

The constraints on the size of the crop rotation or the planning horizon are defined by inequality (1.26)

$$\sum_{i=1}^{n}\sum_{j=1}^{t} X_{rij} \leq T, \tag{1.26}$$

where T – planning horizon for crop rotation, where $T \in \{j\}$ for $j=\overline{1,\ldots,t}$, years.

The consideration of the constraint at the start of crop rotation planning (zero cycle of the project) is given in equation (1.27)

$$X_r \cdot R_0 \cdot j_0 = X_{rj0}, \tag{1.27}$$

where X_r – the allocation of the r predecessor per unit area $\left(X_r \in \{0,1\}\right)$, ha;

R_0 – the initial crop or the predecessor for the first-year crops in the crop rotation $R_0 \in \{1,\ldots,d\}$, in which $R_0 \subseteq I$;

j_0 – the allocation of the predecessor in the j zero (initial) period;

X_{rj0} – the allocation of the r preceding crop in the j initial period of the crop rotation, calculated per unit area $\left(X_{rj0} \in \{0,1\}\right)$, ha.

The constraint on the multiplicity of filled area units (an integer value) is expressed by equation (1.28)

$$\sum_{r=R}^{R}\sum_{i=1}^{n}\sum_{j=t}^{T} X_{rij} = \sum_{r=1}^{d}\sum_{i=R}^{R}\sum_{j=t-1}^{T-1} X_{rij}, \tag{1.28}$$

where R – a set of predecessor cultures from the i set, where $R \supseteq R_r$.

The ratio of crop production per crop within a crop rotation cycle is (1.29)

$$\sum_{r=1}^{d}\sum_{i=1}^{n}\sum_{j=1}^{t} W_i \cdot Y_{ri} \cdot U_{ij} \cdot X_{rij} \geq L_i, \tag{1.29}$$

where L_i – the production volume (output) of the i crop per rotation, q(cwt).

The volume of each type of resource in the crop rotation cycle is shown by the inequality (1.30)

$$\sum_{l=1}^{g}\sum_{r=1}^{d}\sum_{i=1}^{n}\sum_{j=1}^{t} H_{lij} \cdot X_{rij} \leq G_l, \tag{1.30}$$

where H_{lij} – the rate of consumption of the l resource, in accordance with the i cultivation technology for the crop in the j year of the crop rotation, calculated per 1 ha of land (UAH, man-hours, tons, kg of active substance, etc.);

G_l – the volume of the l type of resource planned for use throughout the entire crop rotation period (UAH, man-hours, tons, kg of active substance, etc.).

The limitation on the humus balance across crop rotations explains the unevenness (1.31)

$$\sum_{r=1}^{d}\sum_{i=1}^{n}\sum_{j=1}^{t}\left[\beta_i' \cdot W_i \cdot Y_{ri} \cdot U_{ij} \cdot \left(K_{wi}' + K_{wi}''\right) \cdot X_{rij}\right] - \sum_{r=1}^{d}\sum_{i=1}^{n}\sum_{j=1}^{t}\left[\beta_{iw}'' \cdot X_{rij}\right] \geq 0, \tag{1.31}$$

where β_i' – the humification coefficient of the main and by-products of the i crop;

β_{iw}'' – the humus mineralization level based on the achieved i yield of the crop calculated per 1 ha of plot area, t;

K_{wi}' – the coefficient for calculating the yield of root and post-harvest residues from the main crop of the i crop at the w yield;

K_{wi}'' – the coefficient for calculating the yield of by-products (non-marketable produce) of the i crop at its w yield.

Compliance with crop rotation requirements over time is explained by equation (1.32)

$$U_{ij} = R_i \cdot I_{rj} \cdot e_{ij}, \tag{1.32}$$

where R_i – the allocation of the preceding crop for the i crop, $R_i \in \{1,\dots,d\}$;

I_{rj} – the choice of crop following the r preceding crop in the j year of the crop rotation;

e_{ij} – the rotation (return) of the i crop in the j year of the crop rotation, a binary variable, $e_{ij} \in \{0,1\}$.

A culture is returned if at least one of the requirements is met within the specified time limit for its rotation (1.33):

$$\begin{cases} e_{ij} = 1, \text{if } i_j \geq e_t; \\ e_{ij} = 0, \text{if } i_j < e_t. \end{cases} \tag{1.33}$$

where e_t – the standard return rate of the i crop after t years in the j year of crop rotation.

The independence of the variables is shown by inequality (1.34)

$$X_{rij} \geq 0. \tag{1.34}$$

The LPSolve IDE v.5.5.2.0 software and Python v. 3.9.0 code for non-commercial use were employed to solve the economic-mathematical model of optimizing crop rotations over time and to construct model (reference) variants thereof. The results of the calculations are presented in **Table 1.1**.

Optimizing crop rotation in a dynamic manner in line with the objective shows that, within a three- to seven-field crop rotation system, two crops – namely spring barley and grain maize – proved to be the most effective combinations. The only difference lies in the frequency of their rotation. In any case, grain maize is the main crop in this rotation, where spring barley serves as the preceding crop, although it is more commonly grown.

This is because the gross profit per unit area is quite high for maize, thanks to its high yield and favorable market conditions, compared to other crops, particularly oilseeds. Therefore, in this case as well, the focus on gross profit leads to the selection of crops with the highest profit margins. This result is expected, since the objective function was formulated to establish an economic framework for developing dynamic crop rotation models.

Thus, the formation of a model crop rotation based on economic criteria does not contradict agricultural management practices. Indeed, K. Deininger et al. [55] applied various methods to study the impact of crop rotations and identified a marginality in their composition, where only a few crops predominate.

A description of the directed graphs (trees) based on the optimization data is given in **Table 1.2**.

Table 1.1 Results of crop allocation optimization using the simplex method, depending on the size of crop rotation, based on statistical reports on agricultural production in Ukrainian agricultural enterprises [1, 2]

Crop rotation model (t), years	Computation time, s	Optimal number of agricultural crops	Agricultural crop	Number of repetitions, times	Annual soil loss, t/ha	Erosion risk index for the growing season	Humus balance (+, –), t/ha	Objective function (profit maximization per crop rotation), UAH
3	0.093	2	Spring barley Grain maize	2 1	6.5	0.51	0.61	10050
4	0.094	2	Spring barley Grain maize	2 2	6.8	0.86	0.66	14845
5	0.203	2	Spring barley Grain maize	3 2	6.6	0.65	0.63	17473
6	1.969	2	Spring barley Grain maize	3 3	6.8	0.86	0.66	22268
7	39.92	2	Spring barley Grain maize	4 3	6.6	0.71	0.64	24895

Note: [1] The humus balance does not take into account irreversible losses due to soil erosion.
[2] Erosion losses were calculated on average for slightly eroded land

⬢ **Table 1.2** Results of crop area optimization using the simplex method depending on the crop rotation size, based on directed graph models (Y. Hu's force-directed graph drawing algorithm [56])

Crop rotation model (*t*), years	Allocation by crop type (1 – cereals, 2 – fodder crops), %[1]	Number of nodes (vertices) in the graph, units	Average degree of a graph	Weighted average degree of a graph	Number of short routes	Average route length	Modularity [2]	Graph density	Weight of vertices
3	56.1 43.9	81	0.988	4402.19	222	1.918	13 0.840	0.012	0.0126
4	50.1 49.9	344	0.997	6182.81	1274	2.399	21 0.917	0.003	0.0793
5	53.6 46.1 0.3 (Null)	1435	0.997	7937.15	6729	2.886	56 0.962	0.001	0.399
6	52.2 47.8	5864	1.000	9781.97	33303	3.376	108 0.980	0.000	1.713
7	53.5 46.5	23780	1.000	11610.15	158715	3.869	233 0.991	0.000	6.279

Note: [1] The proportion of optimization results by crop type: the top figure represents cereal crops, the bottom figure represents fodder crops. Exceptions are zero values that were not included in the organization chart (Null).
[2] The modularity of the vertices of a directed graph, where the top number is the number of distinct homogeneous groups and the bottom number is the numerical measure of modularity

Based on the results of solving an economic-mathematical model concerning the effective organization of crop rotations, the models found to be optimal overall included those that increase the intensity of agricultural production and can be classified as non-diversified with a low level of biodiversity. Such crop rotations are still practiced today, and for large-scale commercial production in Ukraine, the ability to choose how to generate quasi-rent and/or excess profits through the use of fertile soils is expected to be a key advantage. However, the model crop rotations (**Table 1.1**) are scientifically sound, having been developed in accordance with the requirements for crop rotation and the influence of the preceding crop on subsequent crops in the rotation. Their organization was carried out according to the step-by-step recommendations proposed by C. Mohler [57], whose study focuses on the narrowing of crop rotations regarding crop selection and repetition – a feature particularly characteristic of short-rotation systems.

We are also observing compliance with the requirement that there should be no humus deficit, as humus levels are in surplus across all scenarios. Of course, this is facilitated by the incorporation of all non-marketable plant biomass, but the simplification of the crop rotation has not disrupted this balance in terms of organic matter. We believe that other issues, including those relating to the storage and sequestration of organic carbon, will be resolved under the appropriate conditions, as indicated by the models.

However, within the scope of our objectives, profit maximization failed to fully meet the regulatory standards for preventing fine-grained soil loss, which exceeds 5 tons per hectare annually. The erosion hazard index for the vegetation period in these rotation models is elevated, which is generally unacceptable. This indicates that, beyond economic expediency, ecological imperatives must be more deeply integrated into the simplex problem constraints to resolve crop rotation challenges effectively.

CONCLUSIONS

This section presents an economic and mathematical justification for the optimization of crop rotations on agricultural enterprises in Ukraine, with a view to ensuring sustainable land management and the resilience of the agricultural sector.

Based on a review of the literature, the resilience of the agricultural sector is defined as its ability to withstand certain external disturbances and global shocks (such as the adverse effects of climate change, market fluctuations and war), and to recover from them through internal adaptive mechanisms and proactive measures. A bibliometric landscape of the global knowledge base on agricultural resilience has been established as a theoretical foundation for ensuring the resilience of Ukraine's agricultural sector. Cluster analysis has shown that the global scientific community's attention is focused primarily on building agricultural resilience to climate change. One of the areas for proactive action should be the optimization of crop rotation schemes for sustainable land management and the resilience of agricultural production.

An economic-mathematical model concerning the design of crop rotations using a combinatorial approach has been formulated. A solution to the problem of crop rotation optimization has been proposed, taking into account the insufficient constraints on environmental conditions. Optimization of crop allocation over time in accordance with the set task has shown that, for a three- to seven-field crop rotation, the best combination is two crops: spring barley and grain maize. Non-diversified crop rotations with low levels of biodiversity, as practiced by large Ukrainian agricultural enterprises, are also optimal models from an economic perspective. However, this does not ensure an adequate level of erosion control, which necessitates the inclusion of environmental criteria in crop rotation optimization models and represents one of the future areas of research.

One of the key regulatory measures for preventing soil degradation is the further refinement of modelling approaches based on optimizing crop rotation patterns and adhering to a number of constraints aimed at minimizing humus deficiency and the risk of soil erosion. The scale of modelling should extend beyond small land parcels and cover larger areas, including within districts and/or regions of Ukraine.

The proposed economic-mathematical model and the results of crop rotation optimization will be useful (1) to policymakers when formulating and implementing agricultural and environmental policies regarding the optimal crop mix in crop rotations, and (2) for managers of agricultural enterprises when planning cropland areas and making management decisions. The findings will be useful to the Ministry of Economy, Environment and Agriculture of Ukraine in the formulation and implementation of soil conservation policies. The findings of the study can be used at various levels of agro-industrial production management when optimizing crop rotations and making management decisions regarding the feasibility of implementing these measures, as well as in the activities of research institutions, advisory services and higher education establishments.

CONFLICT OF INTEREST

The authors declare that they have no conflict of interest regarding this research, including financial, personal, authorship, or other factors that could have influenced the research and the results presented in this article.

USE OF ARTIFICIAL INTELLIGENCE

The authors confirm that no artificial intelligence technologies were used in the creation of this work.

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DATA AVAILABILITY

The manuscript has no associated data.

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AUTHOR CONTRIBUTIONS

Anatolii Kucher: conceptualisation; data curation; formal analysis; funding acquisition; investigation; methodology; project administration; resources; software; supervision; validation; visualisation; writing – original draft; writing – review & editing.

Yevhenii Ulko: conceptualisation; data curation; formal analysis; funding acquisition; investigation; methodology; resources; software; supervision; validation; writing – original draft; writing – review & editing.

Olga Anisimova: conceptualisation; funding acquisition; project administration writing – review & editing.

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